

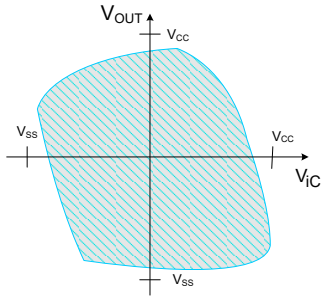
EE 435

Lecture 7:

- High Gain Single-Stage Op Amps

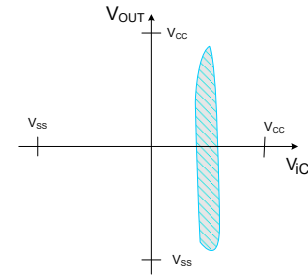
Signal Swing of Single-Stage Op Amp

What type of signal swing is needed ?



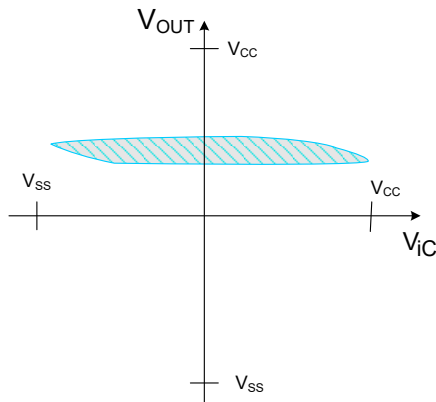
Wide V_{IC} and V_{OUT} range

Expected for catalog parts and overall I/O in many applications



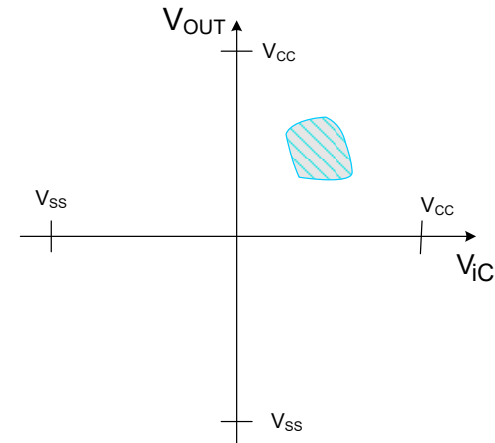
Narrow V_{IC} and wide V_{OUT} range

Acceptable when V_{IC} is fixed



Narrow V_{OUT} and wide V_{IC} range

Acceptable when followed by high-gain stage



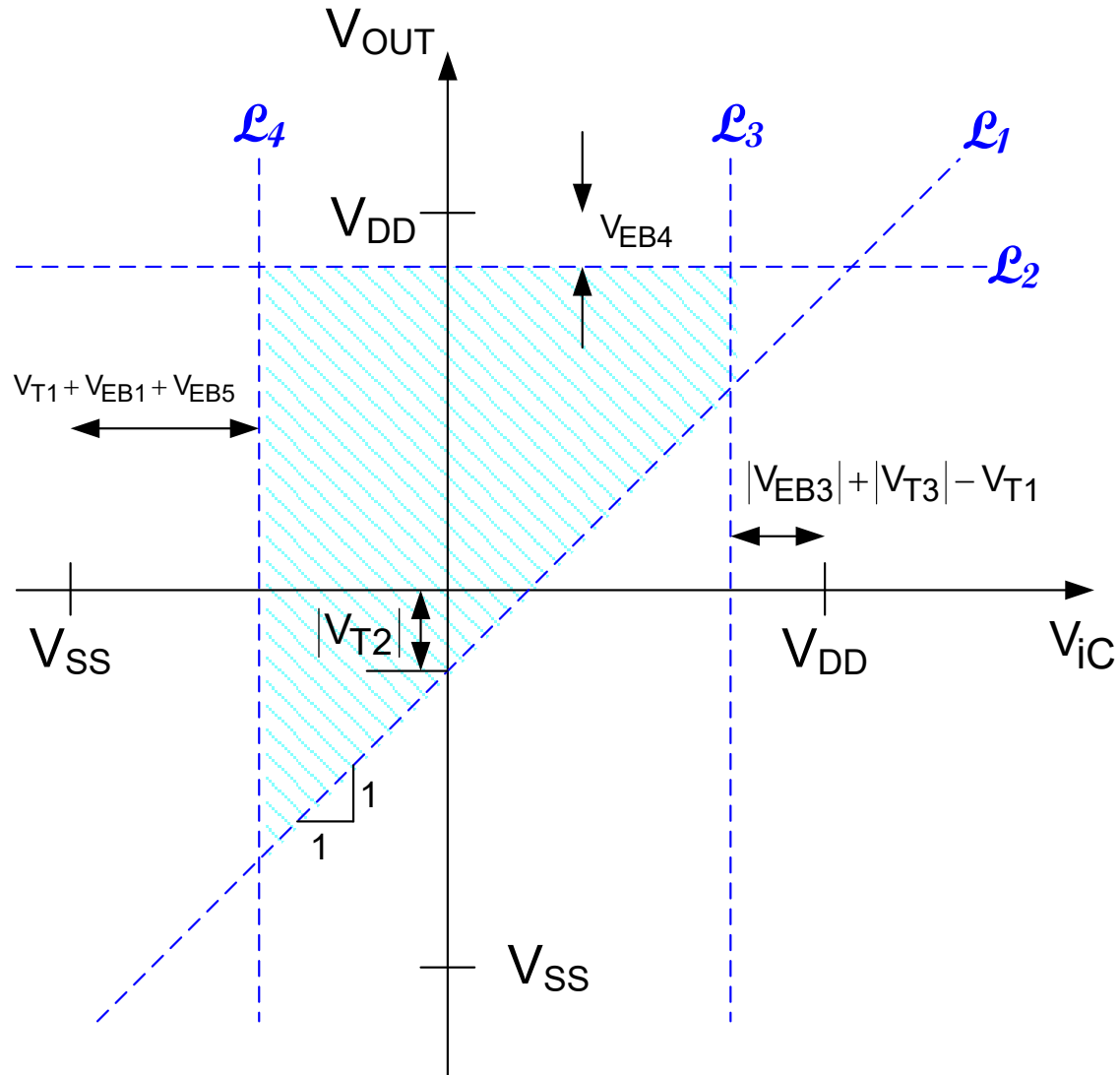
Narrow V_{IC} and V_{OUT} range

Acceptable when V_{IC} fixed and followed by high-gain stage

Review from last lecture:

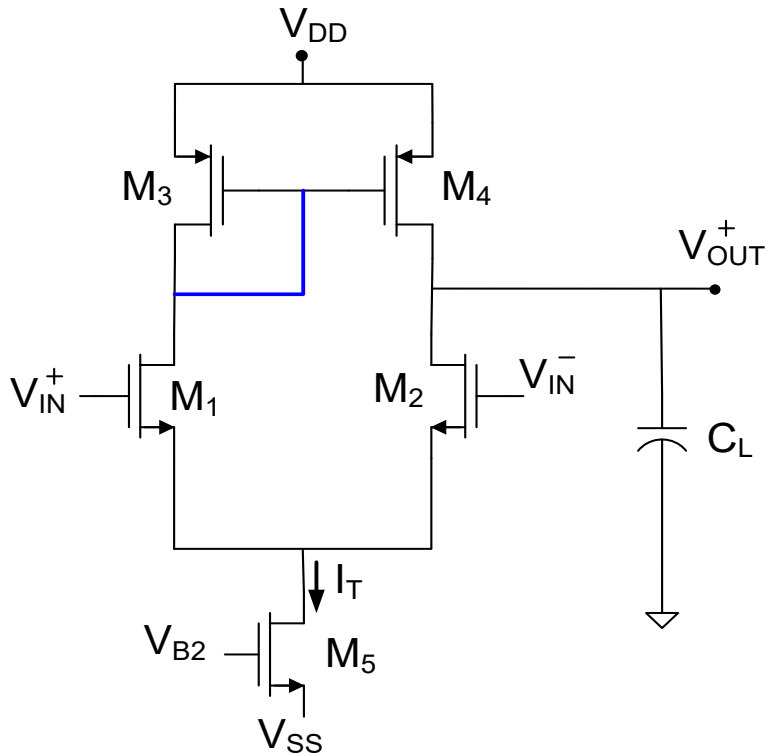
Review from last lecture:

Signal Swing of Single-Stage Op Amp



Review from last lecture:

Design space for single-stage op amp



Performance Parameters in Practical Parameter Domain $\{V_{EB1} V_{EB2} V_{EB5} P\}$:

$$A_0 = \left[\frac{1}{\lambda_1 + \lambda_3} \right] \left(\frac{2}{V_{EB1}} \right)$$

$$GB = \left(\frac{P}{V_{DD} C_L} \right) \left[\frac{1}{V_{EB1}} \right]$$

$$SR = \frac{P}{(V_{DD} - V_{SS}) C_L}$$

$$V_{OUT} < V_{DD} - |V_{EB3}|$$

$$V_{OUT} > V_{ic} - V_{T2}$$

$$V_{ic} < V_{DD} + V_{T1} - |V_{T3}| - |V_{EB3}|$$

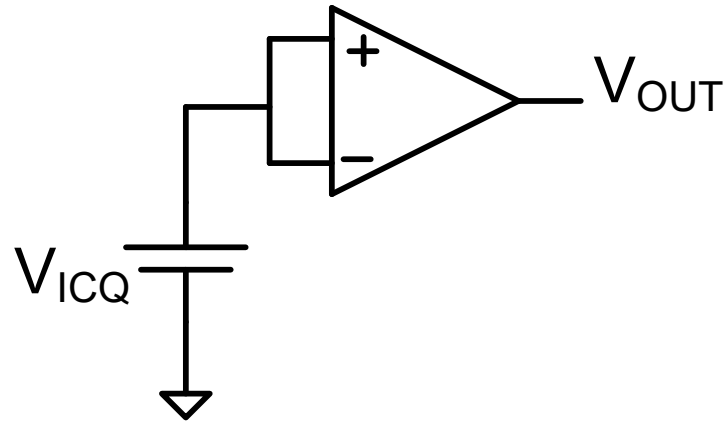
$$V_{ic} > V_{T1} + V_{EB1} + V_{EB5} + V_{SS}$$

Simple Expressions (7) in Practical Parameter Domain

Laboratory Support

Offset Voltage

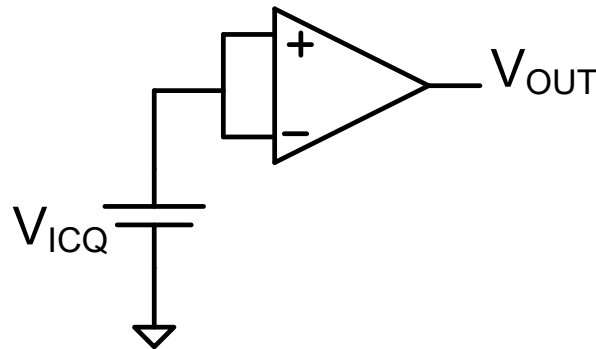
- Systematic Offset Voltage
- Random Offset Voltage



Laboratory Support

Offset Voltage

- Systematic Offset Voltage
- Random Offset Voltage

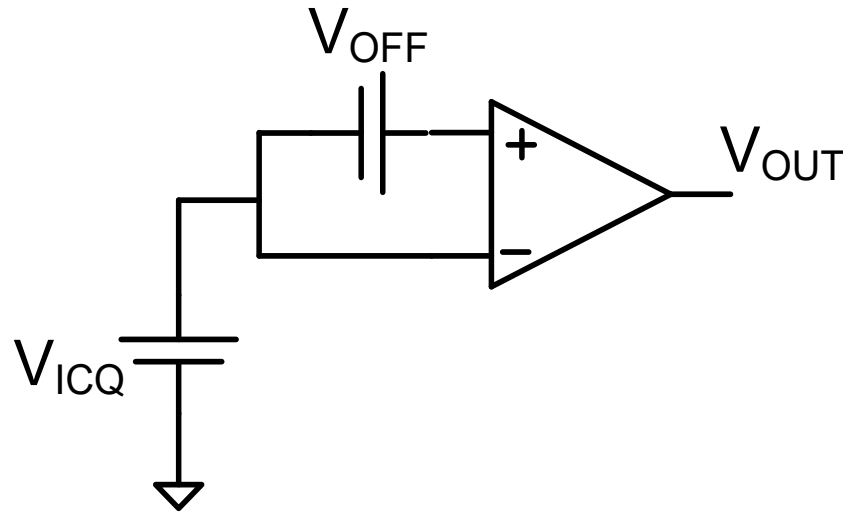


Definition: The output offset voltage is the difference between the desired output and the actual output when $V_{id}=0$ and V_{ic} is the quiescent common-mode input voltage.

$$V_{OUTOFF} = V_{OUT} - V_{OUTDES}$$

Note: V_{OUTOFF} is dependent upon V_{ICQ} although this dependence is usually quite weak and often not specified

Laboratory Support



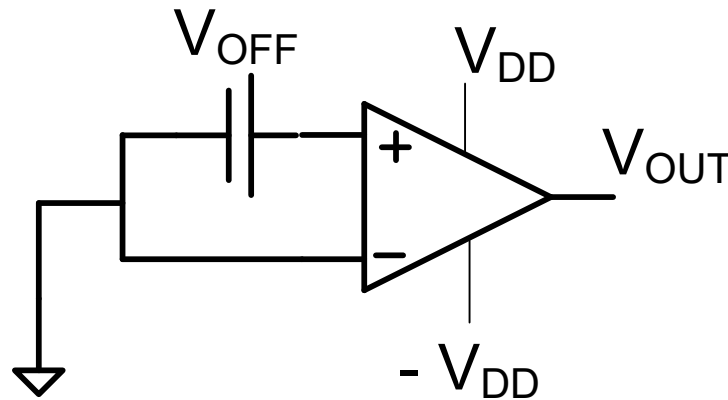
Definition: The input-referred offset voltage is the differential dc input voltage that must be applied to obtain the desired output when V_{ic} is the quiescent common-mode input voltage.

Note: V_{OFF} is usually related to the output offset voltage by the expression

$$V_{OFF} = \frac{V_{OUTOFF}}{A_D}$$

Note: V_{OFF} is dependent upon V_{ICQ} although this dependence is usually quite weak and often not specified

Laboratory Support

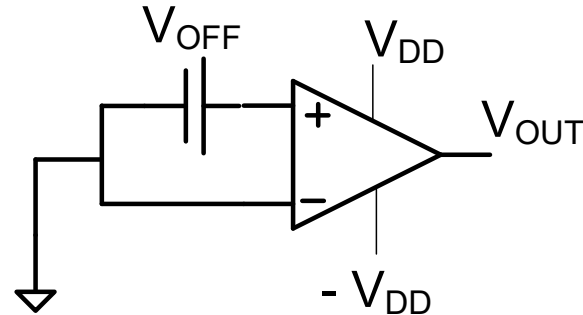


When differential input op amps are biased with symmetric supply voltages, it is generally assumed that the desired quiescent input voltage is 0V and the desired quiescent output voltage is 0V so V_{OFF} is the differential input voltage needed to make $V_{OUT}=0V$.

The input offset voltage is comprised of two parts, a systematic component and a random component

$$V_{OFF} = V_{OFFSYS} + V_{OSR}$$

Laboratory Support



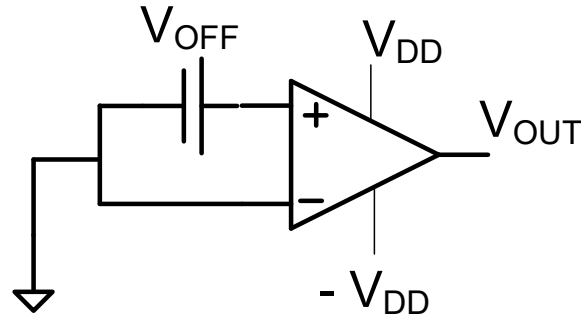
$$V_{OFF} = V_{OFFSYS} + V_{OSR}$$

After fabrication there is no distinction made between V_{OFFSYS} and V_{OSR} and simply V_{OFF} is of concern

V_{OSR} is determined entirely by random variations in component values from their ideal value and will only be seen in a simulation if deviations are intentionally introduced (Monte Carlo Analysis is often used for predicting V_{OSR})

It is expected that V_{OFFSYS} should be small (much smaller than V_{OSR}) and it is the designer's responsibility to make this small

Laboratory Support

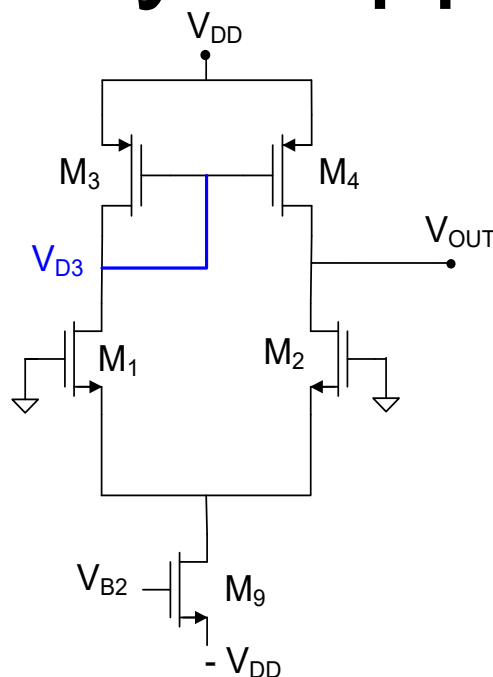


$$V_{OFF} = V_{OFFSYS} + V_{OSR}$$

It is not necessary to make $V_{OFFSYS} = 0$ although this can and is often done by making a minor tweak of matching critical parameters after the design of the op amp is almost complete

V_{OFFSYS} can also be set to 0 by using a degree of freedom of the amplifier design variables but this is generally an unwise use of degrees of freedom (although some textbooks including Martin and Johns in Sec 5.1 do this!)

Laboratory Support

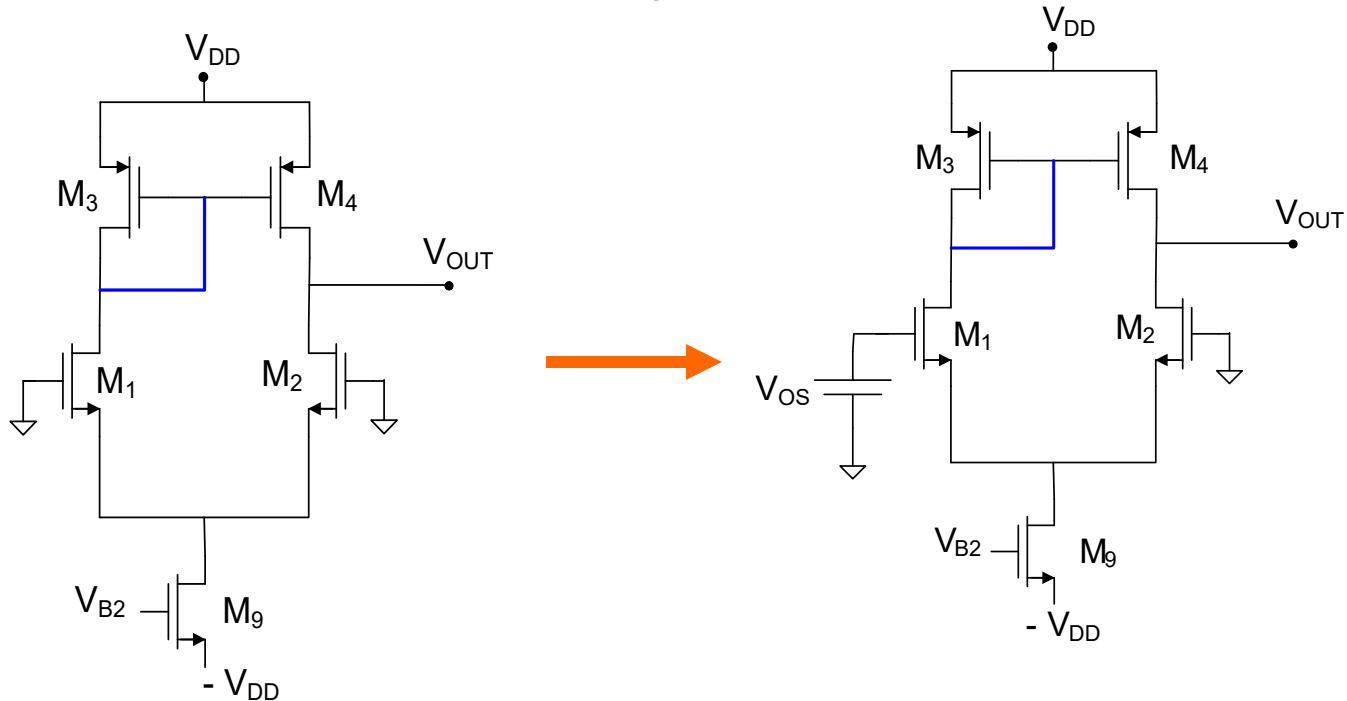


(If no mismatch is introduced, will be seeing only effects of systematic offset)

By symmetry, to force $V_{OUT} = 0$, it is necessary to have $V_{D3} = 0$

- Making $V_{D3} = 0$ sets $|V_{EB3}| = V_{DD} + V_{Tp}$ and results in the use of one degree of freedom!
- Making V_{EB3} so large will severely limit the voltage swing at V_{OUT}
- This shows why it is not wise to use a degree of freedom to make desired output voltage 0

Laboratory Support

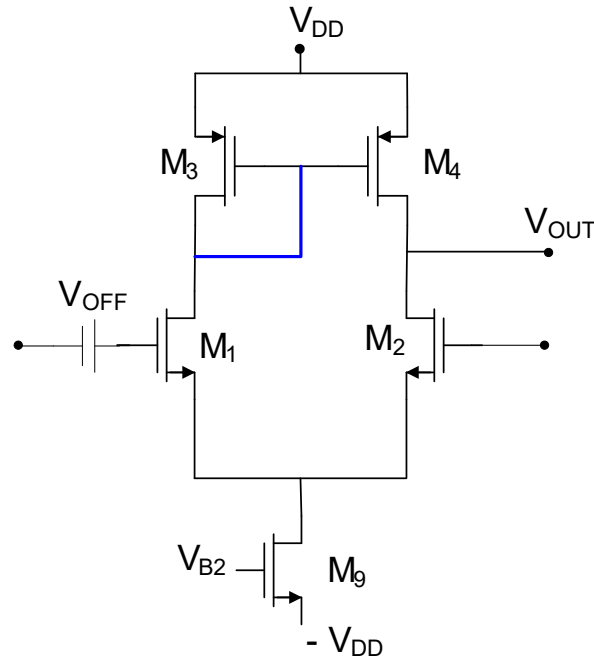


Can sweep a voltage in simulator at gate of M_1 to make $V_{OUT} = V_{OUT_DESIRED}$

The value of V_{OS} that makes $V_{OUT} = V_{OUT_DES}$ is the systematic offset voltage

Can simply add the systematic offset voltage to input throughout rest of the design phase and then remove after design is complete or tweak at end of design to eliminate systematic offset.

Laboratory Support



Usually V_{OFF} will change if changes in any design variables are made so re-simulation will be needed to get the correct value of V_{OFF}

If V_{OFF} is not included, ac simulation of open-loop amplifier will usually not give desired results because small-signal models will be developed in simulator at incorrect operating point (often even in incorrect region of operation)

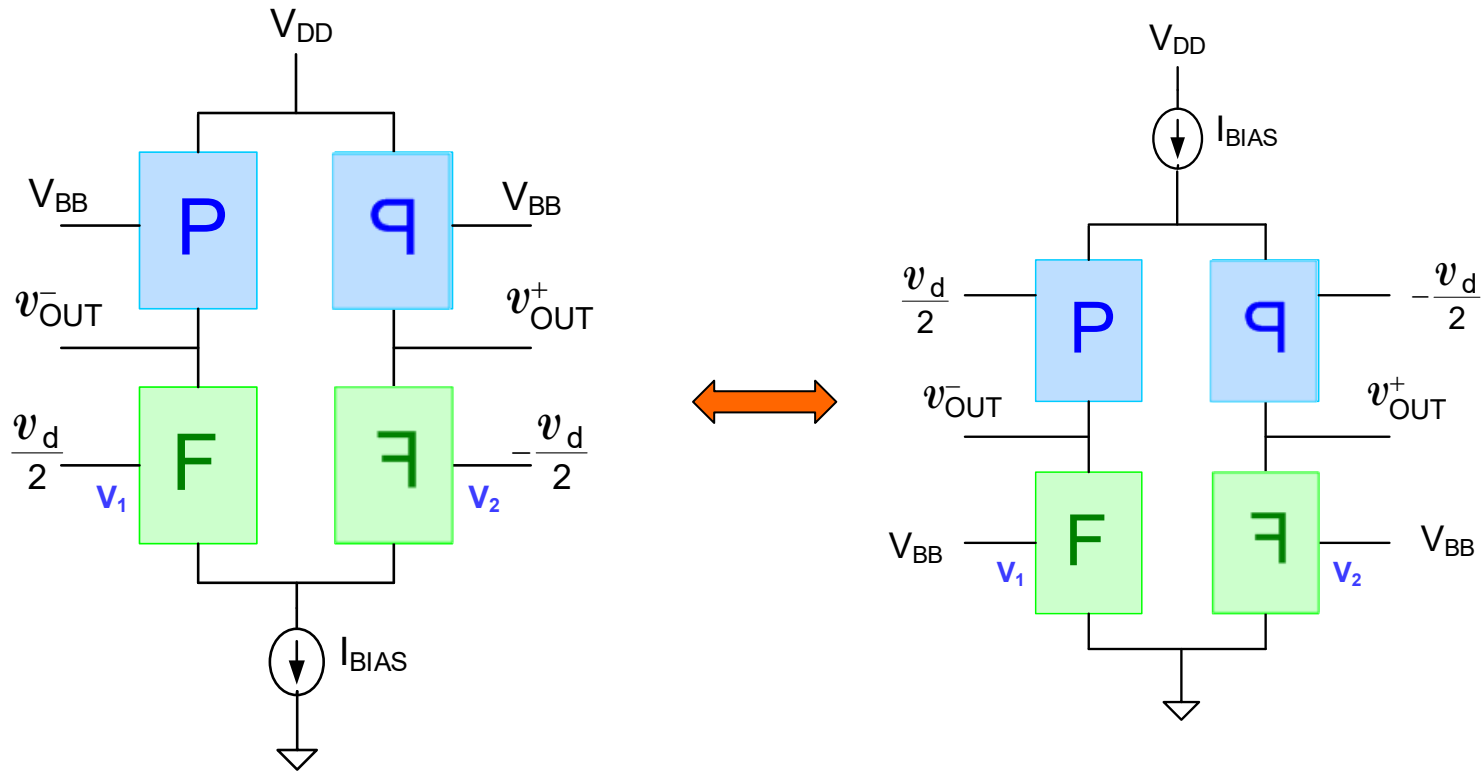
Alternative is to do ac simulations by embedding op amp into a FB configuration that will inherently compensate for offset voltage but issue of compensation must be addressed for amplifiers with two or more poles

Where we are at:

Basic Op Amp Design

- Fundamental Amplifier Design Issues
- • Single-Stage Low Gain Op Amps
- Single-Stage High Gain Op Amps
- Other Basic Gain Enhancement Approaches
- Two-Stage Op Amp

Inputs into Counterpart Circuit or Quarter Circuit

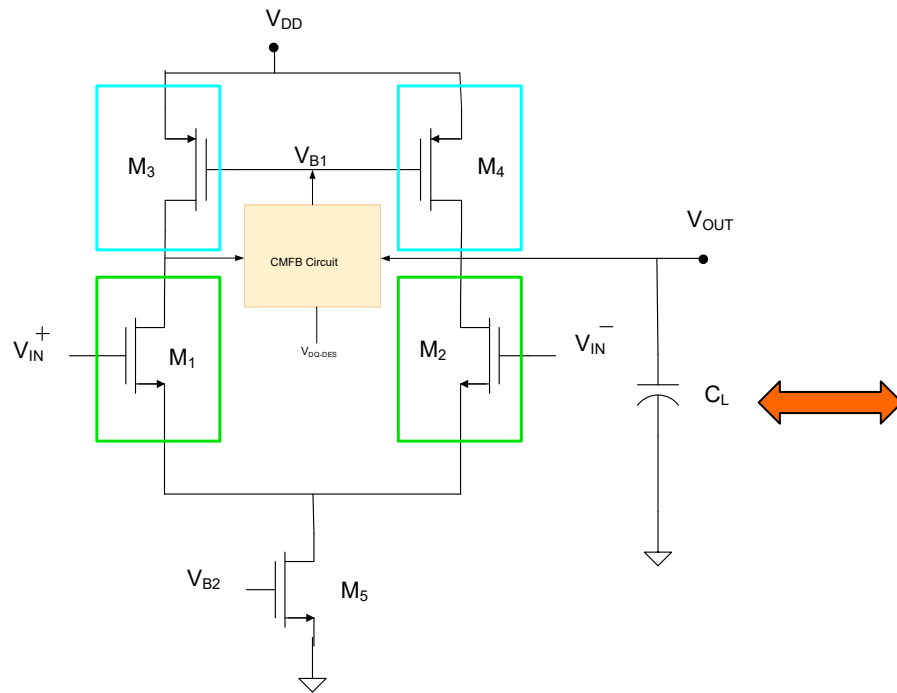


Gain, BW, and GB expressions identical

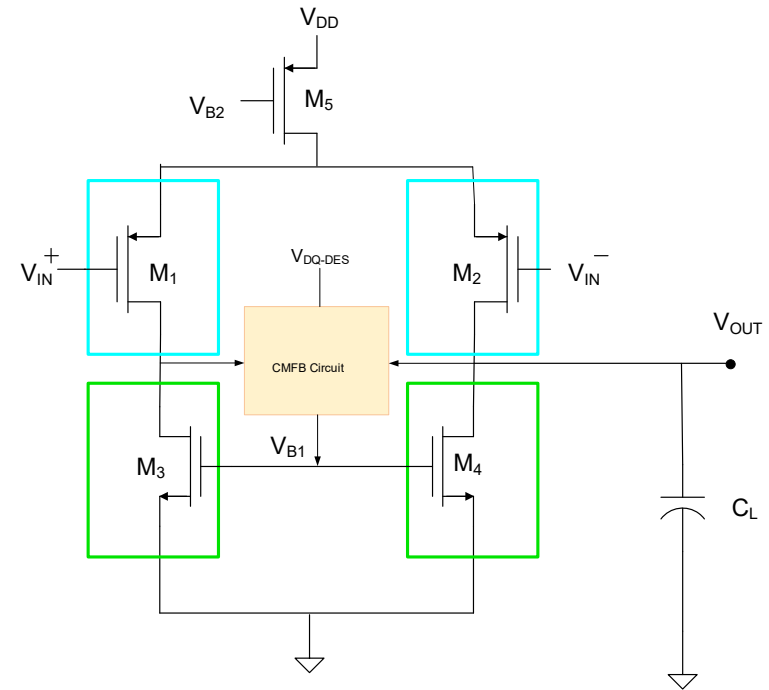
- This is a general concept not related to what type of quarter circuit is used
- Performance may be different because n-channel and p-channel performance different

Inputs into Counterpart Circuit or Quarter Circuit for single-transistor quarter circuit

5T Op Amp with n-ch inputs



5T Op Amp with p-ch inputs



Gain, BW, and GB expressions identical

- Performance may be different because n-channel and p-channel performance different
- Both are widely used

Single-stage op amps

Question – is the gain achievable with the single-stage low-gain op amps using a single MOS transistor as a quarter circuit adequate?

$$A_{v0} = \left[\frac{1}{\lambda_1 + \lambda_3} \right] \left(\frac{1}{V_{EB1}} \right)$$

If $\lambda_1 = \lambda_3 = .01V^{-1}$ and $V_{EB1} = .15V$, then

$$A_{v0} \approx \frac{1}{(.01 + .01)} \frac{1}{0.15} = 333$$

or, in db, $A_{v0db} = 20 \log_{10} 333 = 50db$

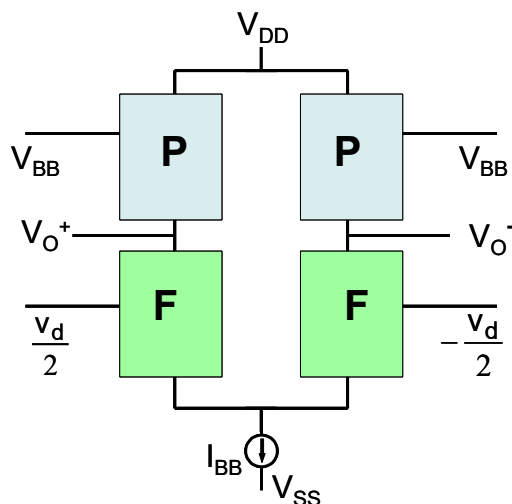
This is inadequate for many applications !

What can be done about it ?

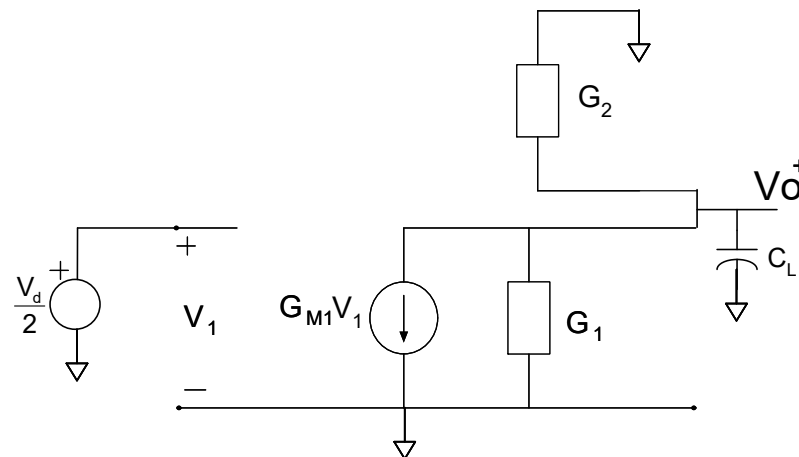


Recall from a previous lecture:

Determination of op amp characteristics from quarter circuit characteristics



Small signal differential half-circuit



$$A_V = \frac{V_{O^+}}{V_d} = \frac{-\frac{G_{M1}}{2}}{sC_L + G_1 + G_2}$$



$$A_{VO} = \frac{-G_{M1}}{2(G_1 + G_2)}$$

$$BW = \frac{G_1 + G_2}{C_L}$$

$$GB = \frac{G_{M1}}{2C_L}$$

Recall from a previous lecture:

Single-Stage High Gain Op Amps

How can the gain of the op amp be increased?

Recall from Quarter-Circuit Concept

$$A_{VO} = \frac{1}{2} \frac{-G_{M1}}{G_1 + G_2}$$

A possible strategy :

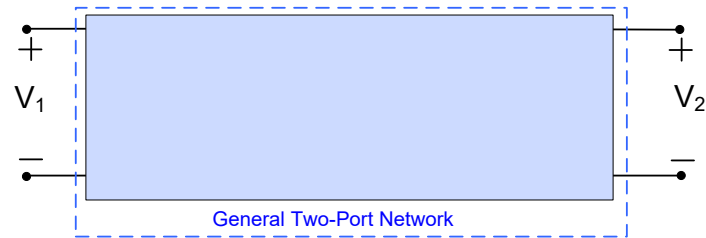
Increase G_{M1} or Decrease G_1 (and G_2)
in Quarter Circuit or Both

Recall from a previous lecture:

Single-Stage High-Gain Op Amps

- If the output conductance can be decreased without changing the transconductance, the gain can be enhanced
- Will concentrate on quarter-circuits and extend to op amps

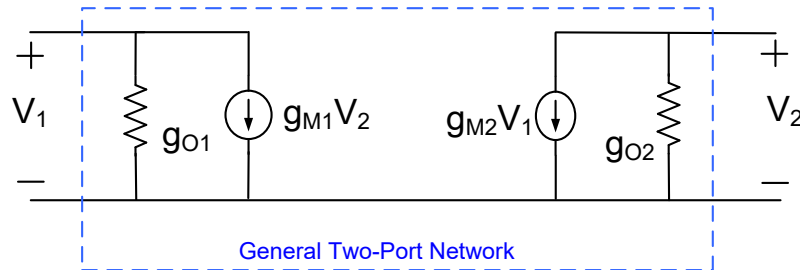
Determination of 2-port parameters



Determination of $\{g_{o1}, g_{o2}, g_{M1}, g_{M2}\}$

Method 1 Open-Short Termination Approach

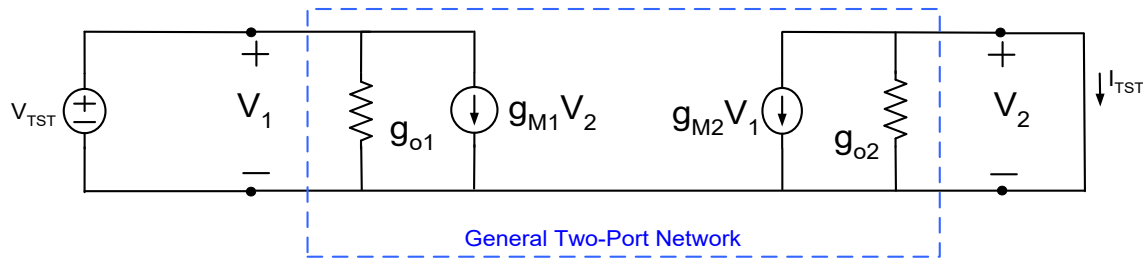
Method 2 Load Termination Approach



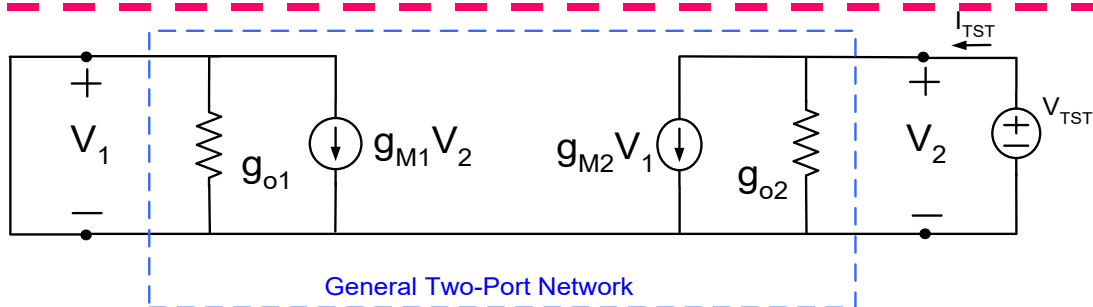
Determination of 2-port parameters

Determination of $\{g_{o1}, g_{o2}, g_{M1}, g_{M2}\}$

Method 1 Open-Short Termination Approach



$$g_{M2} = -\frac{I_{TST}}{V_{TST}}$$



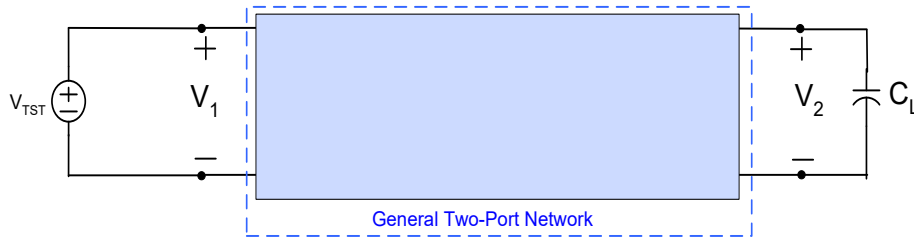
$$g_{o2} = \frac{I_{TST}}{V_{TST}}$$

By structural symmetry, repeat to obtain g_{M1} and g_{o1}

Determination of 2-port parameters

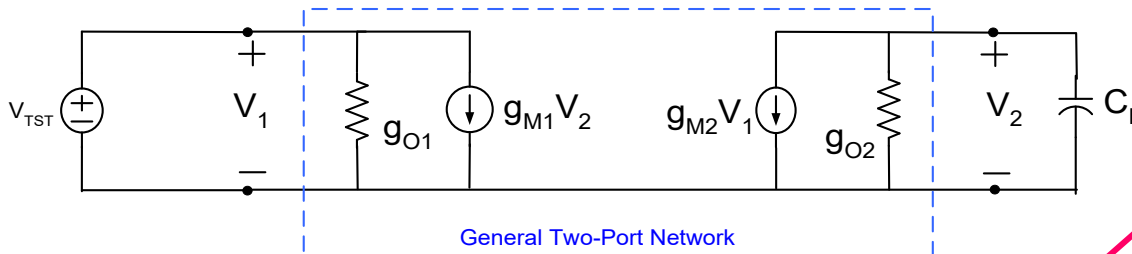
Determination of $\{g_{o1}, g_{o2}, g_{M1}, g_{M2}\}$

Method 2 Load Termination Approach



Since first-order express the gain $A(s)$ in form

$$A(s) = \frac{a_0}{sC_L + b_0}$$



observe

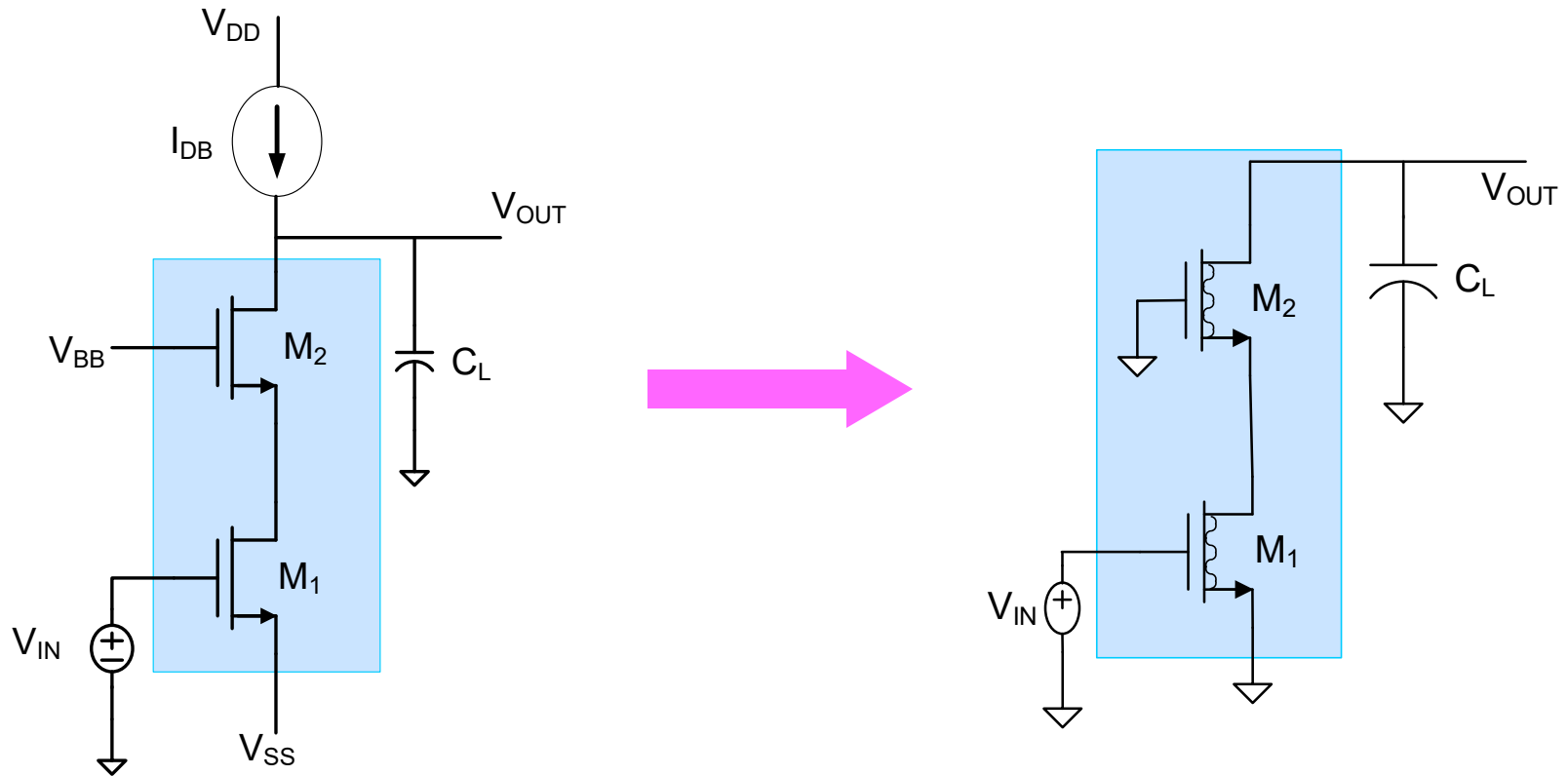
$$V_2(g_{O2} + sC_L) + g_{M2}V_{TST} = 0$$

$$A(s) = \frac{V_2(s)}{V_{TST}(s)} = -\frac{g_{M2}}{sC_L + g_{O2}}$$

(must express with coefficient of s in den equal to C_L)

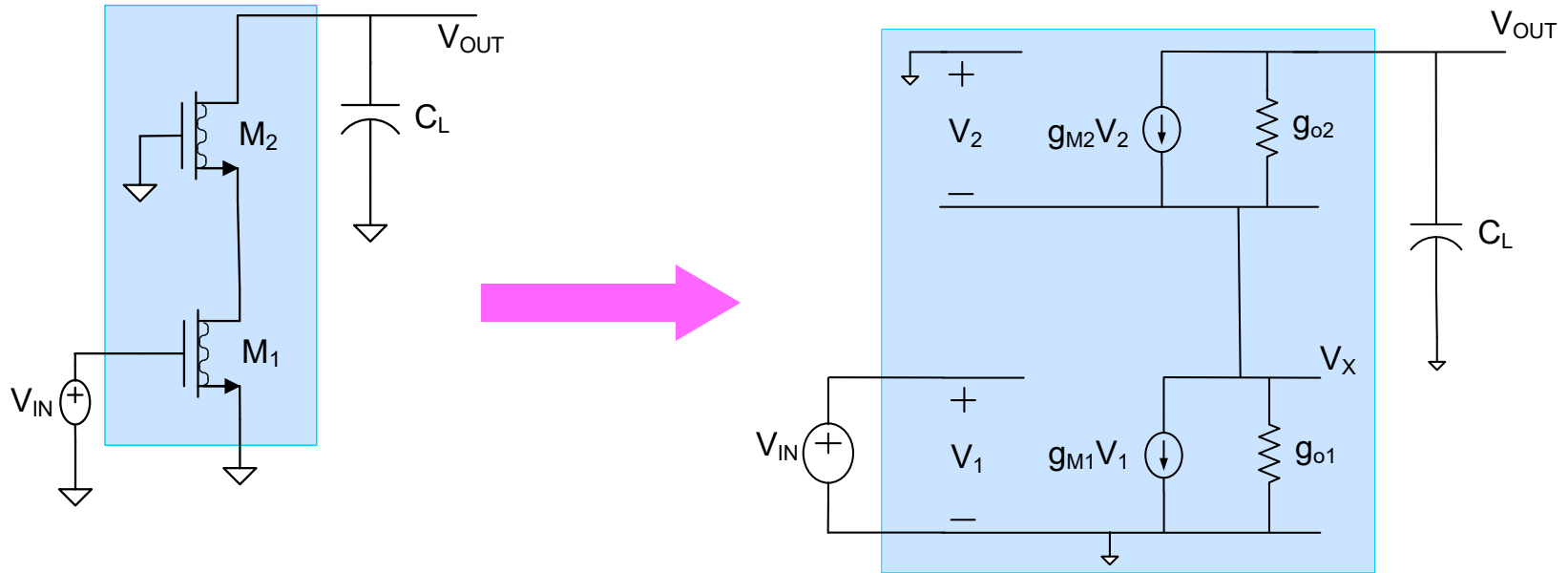
Background

Recall Cascode Amplifier (dc current source bias)



Background

Analysis of Cascode Amplifier (dc current source bias)

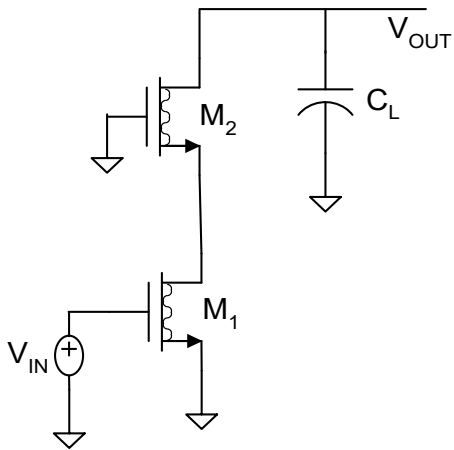


$$\left. \begin{aligned} \mathbf{V}_{\text{OUT}}(\mathbf{g}_{o2} + \mathbf{sC}_L) + \mathbf{g}_{m2} \mathbf{V}_2 &= \mathbf{V}_X \mathbf{g}_{o2} \\ \mathbf{V}_X(\mathbf{g}_{o1} + \mathbf{g}_{o2}) + \mathbf{g}_{m1} \mathbf{V}_1 - \mathbf{g}_{m2} \mathbf{V}_2 &= \mathbf{V}_{\text{OUT}} \mathbf{g}_{o2} \\ \mathbf{V}_2 &= -\mathbf{V}_X \\ \mathbf{V}_1 &= \mathbf{V}_{\text{IN}} \end{aligned} \right\}$$

V_X, V_1 and V_2 can be eliminated from these 4 equations

Background

Analysis of Cascode Amplifier



$$\left. \begin{aligned} \mathbf{V}_{OUT}(\mathbf{g}_{o2} + \mathbf{s}C_L) + \mathbf{g}_{m2}\mathbf{V}_2 &= \mathbf{V}_X\mathbf{g}_{o2} \\ \mathbf{V}_X(\mathbf{g}_{o1} + \mathbf{g}_{o2}) + \mathbf{g}_{m1}\mathbf{V}_1 - \mathbf{g}_{m2}\mathbf{V}_2 &= \mathbf{V}_{OUT}\mathbf{g}_{o2} \\ \mathbf{V}_2 &= -\mathbf{V}_X \\ \mathbf{V}_1 &= \mathbf{V}_{IN} \end{aligned} \right\}$$

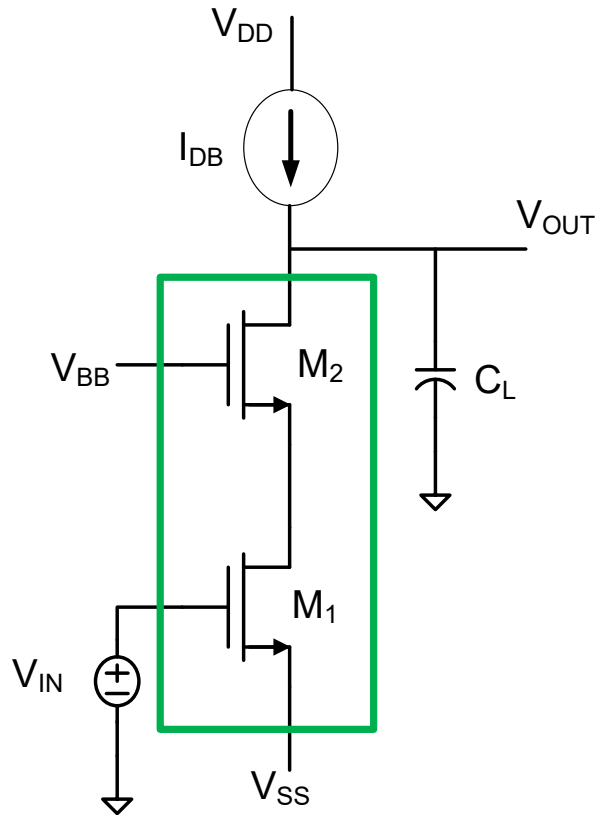
$$\left. \begin{aligned} \mathbf{V}_{OUT}(\mathbf{g}_{o2} + \mathbf{s}C_L) - \mathbf{g}_{m2}\mathbf{V}_X &= \mathbf{V}_X\mathbf{g}_{o2} \\ \mathbf{V}_X(\mathbf{g}_{o1} + \mathbf{g}_{o2}) + \mathbf{g}_{m1}\mathbf{V}_{IN} + \mathbf{g}_{m2}\mathbf{V}_X &= \mathbf{V}_{OUT}\mathbf{g}_{o2} \end{aligned} \right\}$$

$$\frac{V_{OUT}}{V_{IN}} = \frac{-g_{m1}(g_{o2} + g_{m2})}{sC_L(g_{o1} + g_{o2} + g_{m2}) + g_{o1}g_{o2}} \approx \frac{-g_{m1}g_{m2}}{sC_Lg_{m2} + g_{o1}g_{o2}}$$

$$\frac{V_{OUT}}{V_{IN}} \approx \frac{-g_{m1}}{sC_L + g_{o1}\left(\frac{g_{o2}}{g_{m2}}\right)}$$

g_{MEQ}
 g_{OEQ}

High output impedance quarter-circuits



Cascode Amplifier

$$G = g_{\text{OEQ}} \cong g_{\text{O1}} \left[\frac{g_{\text{o2}}}{g_{\text{m2}}} \right]$$

$$G_{\text{M}} = g_{\text{mEQ}} \cong g_{\text{m1}}$$

- **Transconductance appears to be unchanged** (from basic single-device structure)
- **Output conductance appears to have been decreased !**

$$A_{\text{v}}(s) \cong \frac{-g_{\text{m1}}}{sC_{\text{L}} + g_{\text{o1}} \left[\frac{g_{\text{o2}}}{g_{\text{m2}}} \right]}$$

$$A_{\text{v0}} \cong \left(\frac{g_{\text{m1}}}{g_{\text{o1}}} \right) \left[\frac{g_{\text{m2}}}{g_{\text{o2}}} \right]$$

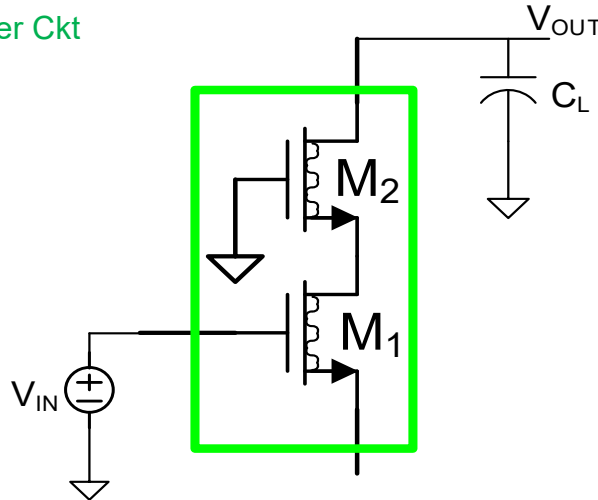
$$GB \cong \frac{g_{\text{m1}}}{C_{\text{L}}}$$

But must verify in the practical parameter domain to be sure!

Comparison of Basic Amplifier and Cascode Amplifier

$$\frac{V_{OUT}}{V_{IN}} = \frac{-G_m}{sC_L + G}$$

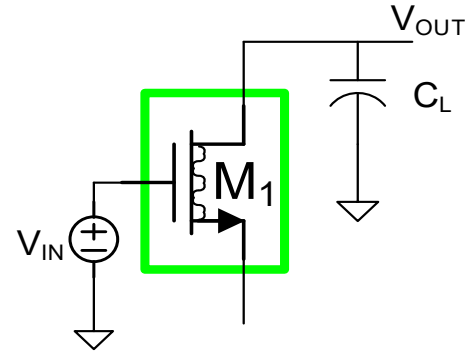
General form for any Quarter Ckt



$$\frac{V_{OUT}}{V_{IN}} \approx \frac{-g_{m1}}{sC_L + g_{o1} \left(\frac{g_{o2}}{g_{m2}} \right)}$$

$$G = g_{OEQ} \cong g_{O1} \left[\frac{g_{O2}}{g_{m2}} \right]$$

$$G_M = g_{mEQ} \cong g_{m1}$$



$$\frac{V_{OUT}}{V_{IN}} = \frac{-g_{m1}}{sC_L + g_{o1}}$$

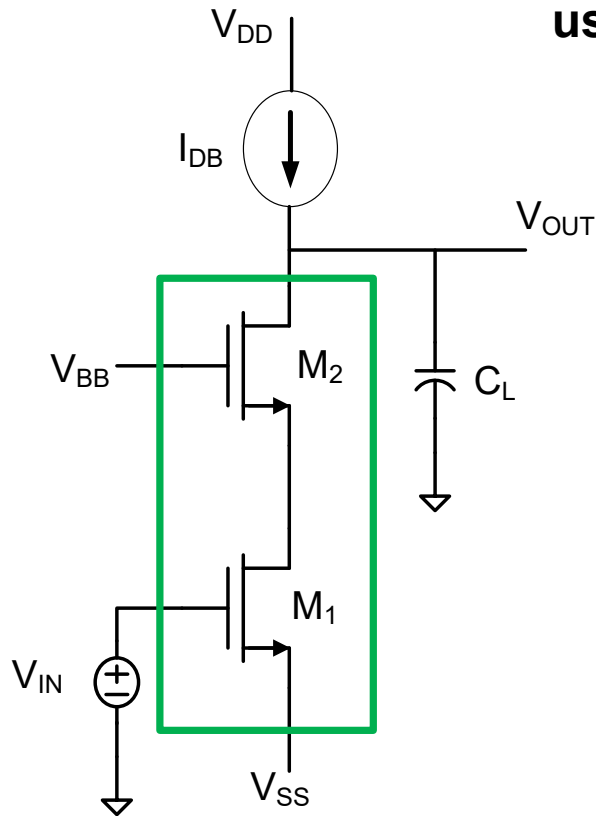
$$G = g_{O1}$$

$$G_M = g_{m1}$$

High output impedance quarter-circuits

How does this compare with previous amplifier using single transistor as quarter circuit?

Cascode amplifier quarter circuit:



Cascode Amplifier

$$A_{V0} \cong \left(\frac{g_{m1}}{g_{o1}} \right) \left[\frac{g_{m2}}{g_{o2}} \right] \quad GB \cong \frac{g_{m1}}{C_L}$$

$$A_{V0} \cong \left[\frac{2}{\lambda_1 V_{EB1}} \right] \cdot \left[\frac{2}{\lambda_2 V_{EB2}} \right]$$

$$GB = \left[\frac{2P}{V_{DD} C_L} \right] \cdot \left[\frac{1}{V_{EB1}} \right]$$

Single transistor quarter circuit:

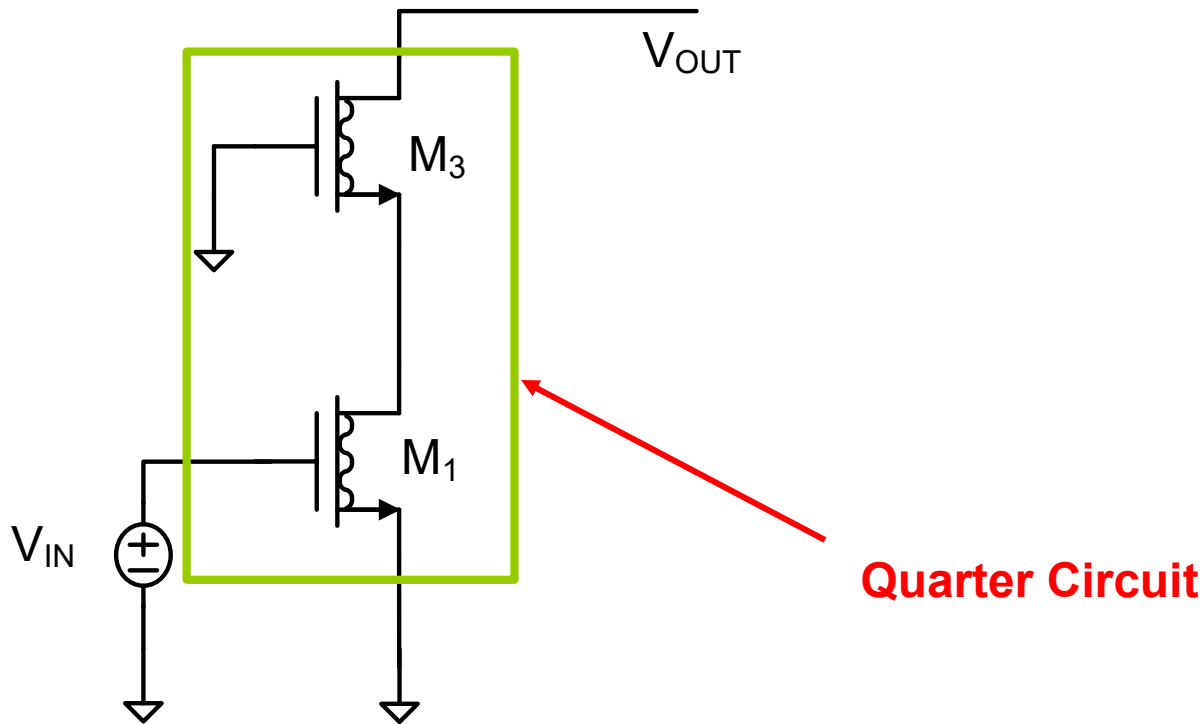
$$A_{V0} = \left[\frac{2}{\lambda V_{EB}} \right]$$

$$GB = \left(\frac{2P}{V_{DD} C_L} \right) \cdot \left(\frac{1}{V_{EB}} \right)$$

Substantial increase in dc gain

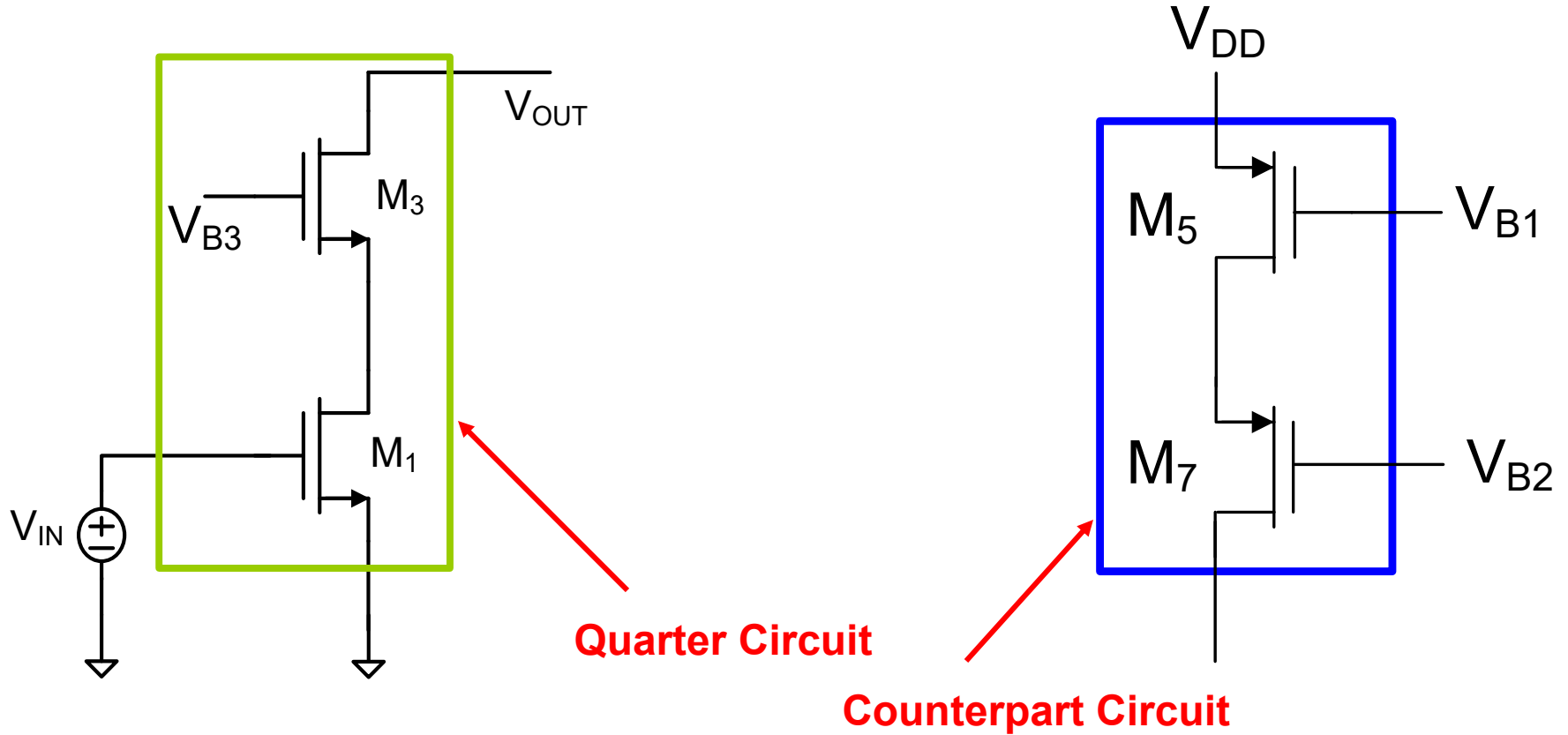
No improvement in GB but also no deterioration in GB !

High output impedance quarter-circuits



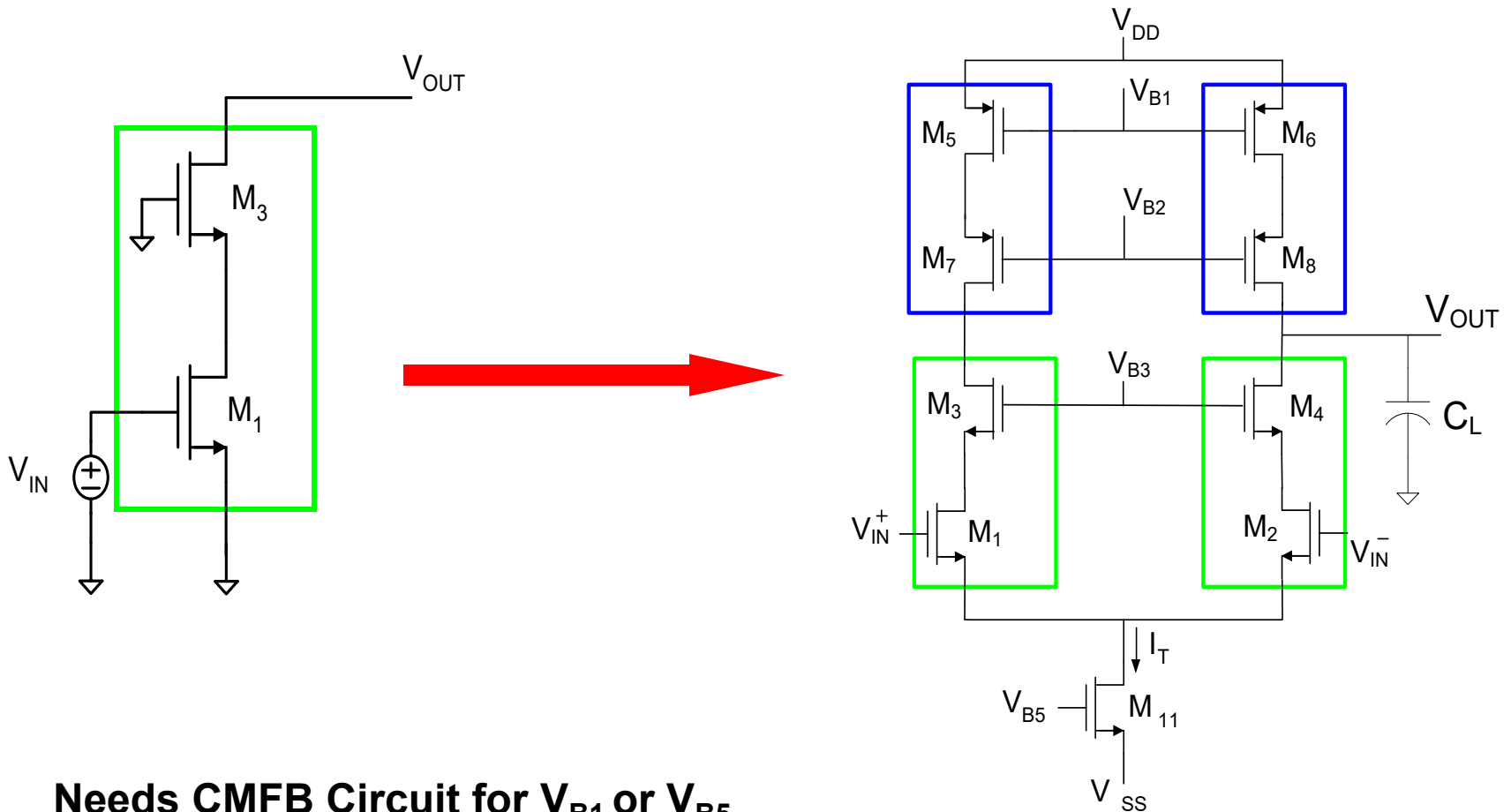
**Cascode Amplifier
(small-signal equiv)**

High output impedance quarter-circuits



Cascode Amplifier

Telescopic Cascode Op Amp

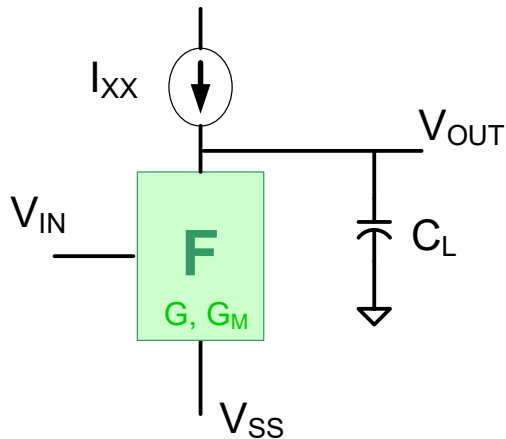


Needs CMFB Circuit for V_{B1} or V_{B5}
Either single-ended or differential outputs
Can connect counterpart as current mirror to eliminate CMFB

Recall:

Determination of op amp characteristics from quarter circuit characteristics (for single-ended gain)

Small signal Quarter Circuit



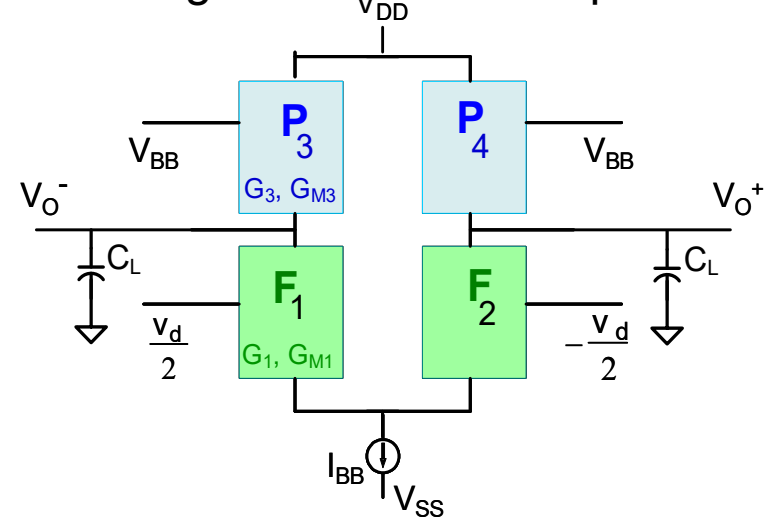
$$A_{V_{OQC}} = -\frac{G_M}{G}$$

$$BW = \frac{G}{C_L}$$

$$GB = \frac{G_M}{C_L}$$

$$A(s) = \frac{-G_M}{sC_L + G}$$

Small signal differential amplifier



$$A_{V_0} = \frac{-\frac{G_{M1}}{2}}{(G_1 + G_3)}$$

$$BW = \frac{G_1 + G_3}{C_L}$$

$$GB = \frac{G_{M1}}{2C_L}$$

$$A(s) = \frac{-\frac{G_{M1}}{2}}{sC_L + G_1 + G_3}$$

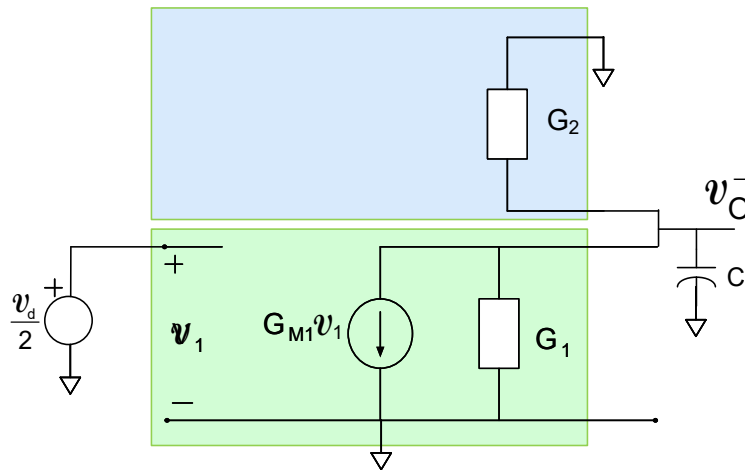


Note: Factor of 4 reduction of single-ended gain (for $G_1 = G_2$)

Recall equivalently:

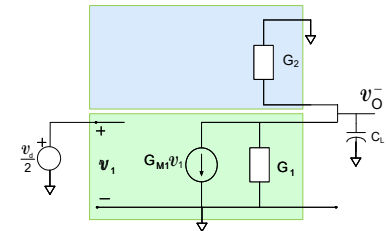
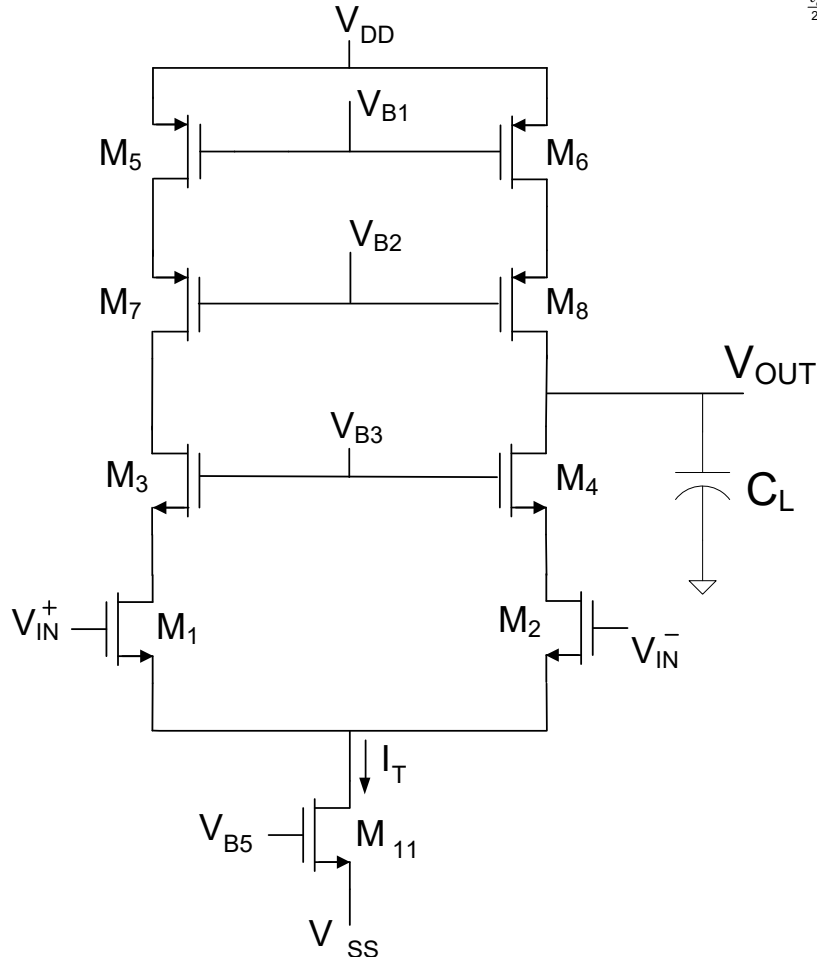
Recall from earlier lecture:

-- The “differential” gain --



$$A_{Vd} = \frac{v_o^-}{v_d} = \frac{-\frac{G_{M1}}{2}}{sC_L + G_1 + G_2}$$

Telescopic Cascode Op Amp



$$A_{Vd} = \frac{v_o^-}{v_d} = \frac{-\frac{G_{M1}}{2}}{sC_L + G_1 + G_2}$$

Single-ended operation

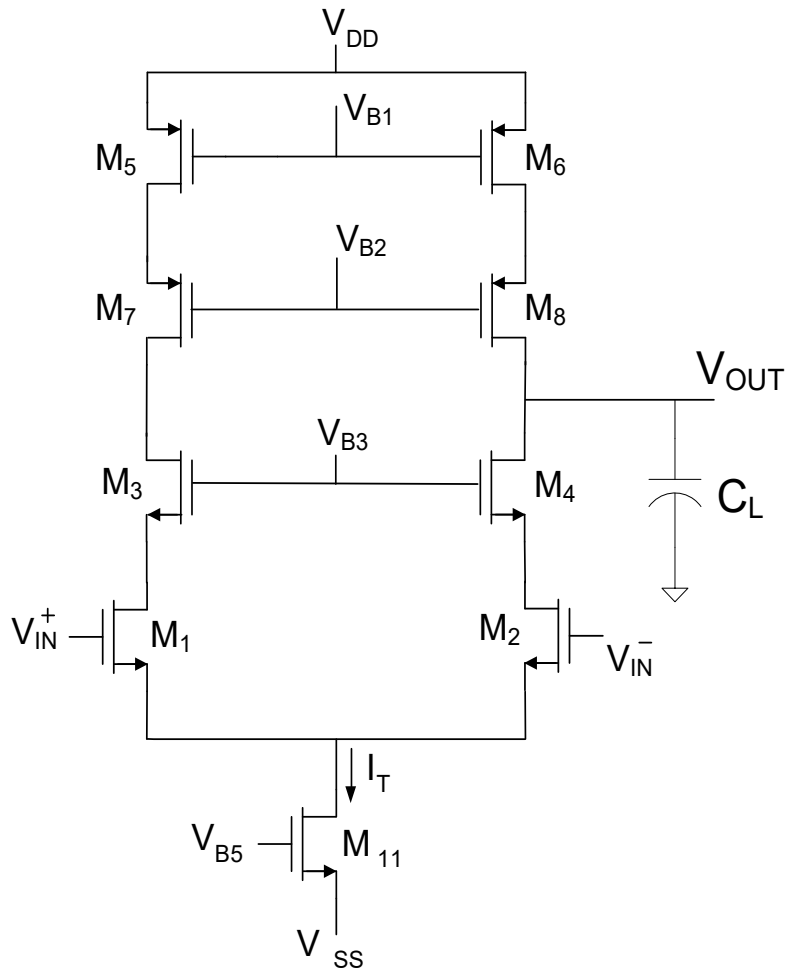
$$G_1 = g_{OQC} = \underline{\hspace{10em}}$$

$$G_2 = g_{OCC} = \underline{\hspace{10em}}$$

$$G_{M1} = g_{mQC} = \underline{\hspace{10em}}$$

Telescopic Cascode Op Amp

Single-ended operation

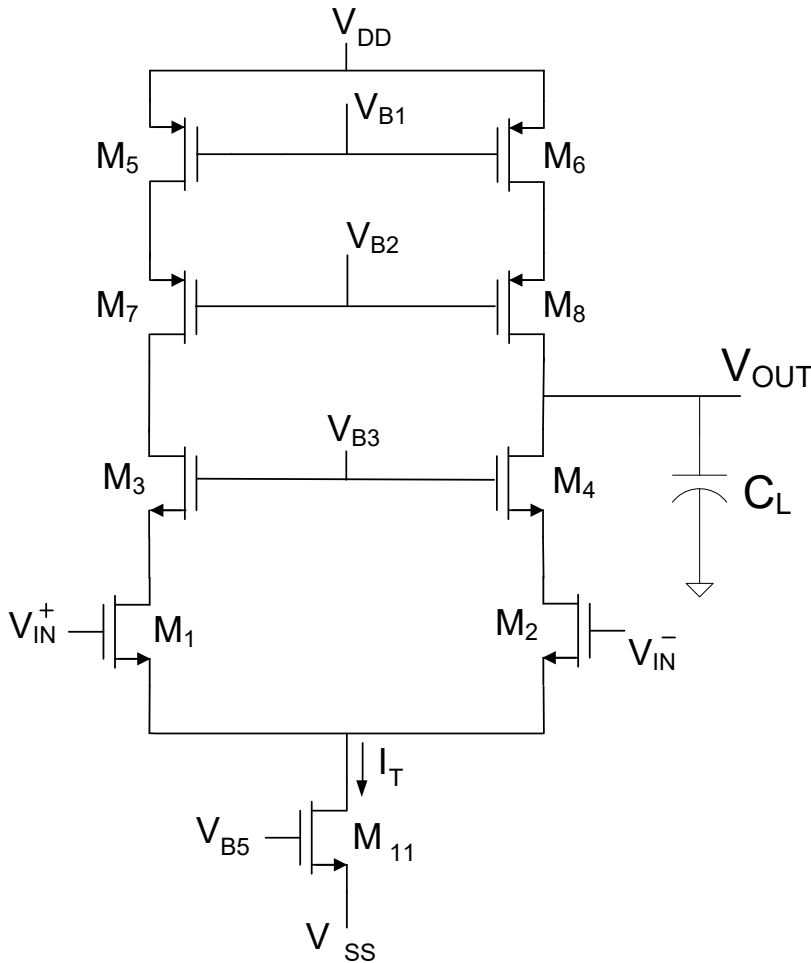


$$A_d(s) = \frac{-\frac{g_{m1}}{2}}{sC_L + g_{o1} \frac{g_{o3}}{g_{m3}} + g_{o5} \frac{g_{o7}}{g_{m7}}}$$

$$A_o = \frac{-\frac{g_{m1}}{2}}{g_{o1} \frac{g_{o3}}{g_{m3}} + g_{o5} \frac{g_{o7}}{g_{m7}}}$$

$$GB = \frac{g_{m1}}{2C_L}$$

Telescopic Cascode Op Amp



(CMFB circuit not shown)

Single-ended operation

$$A_d(s) = \frac{-\frac{g_{m1}}{2}}{sC_L + g_{o1} \frac{g_{o3}}{g_{m3}} + g_{o5} \frac{g_{o7}}{g_{m7}}}$$

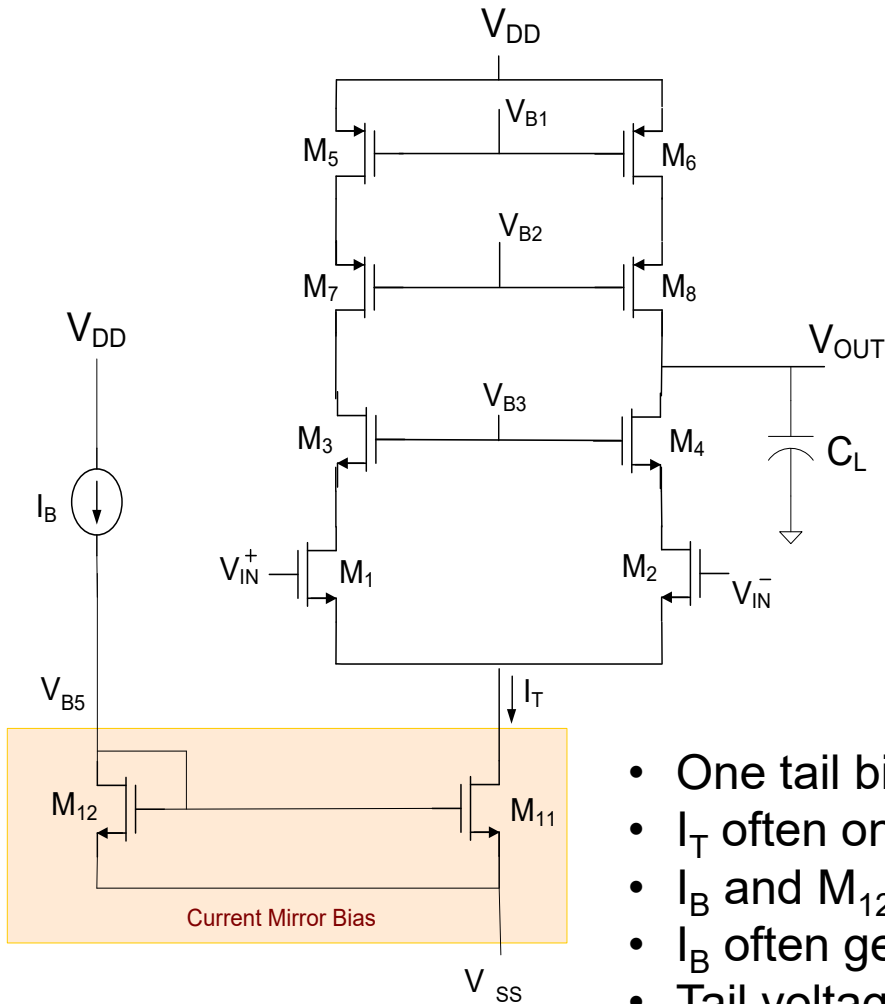
$$A_0 = \frac{-\frac{g_{m1}}{2}}{g_{o1} \frac{g_{o3}}{g_{m3}} + g_{o5} \frac{g_{o7}}{g_{m7}}}$$

$$GB = \frac{g_{m1}}{2C_L}$$

- Large improvement in A_0
- No change in GB

This circuit is widely used !!

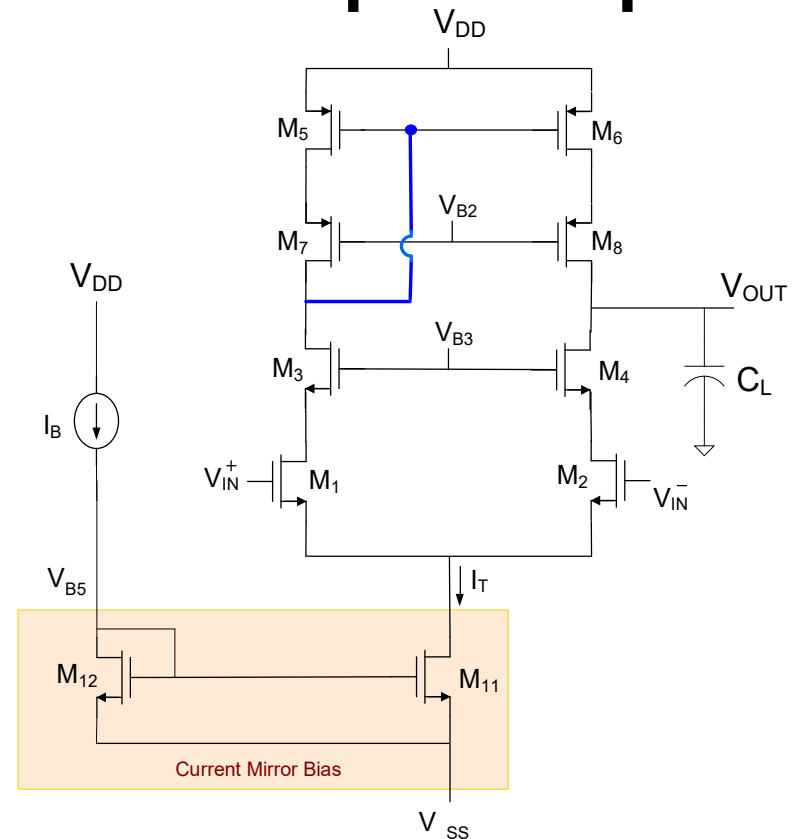
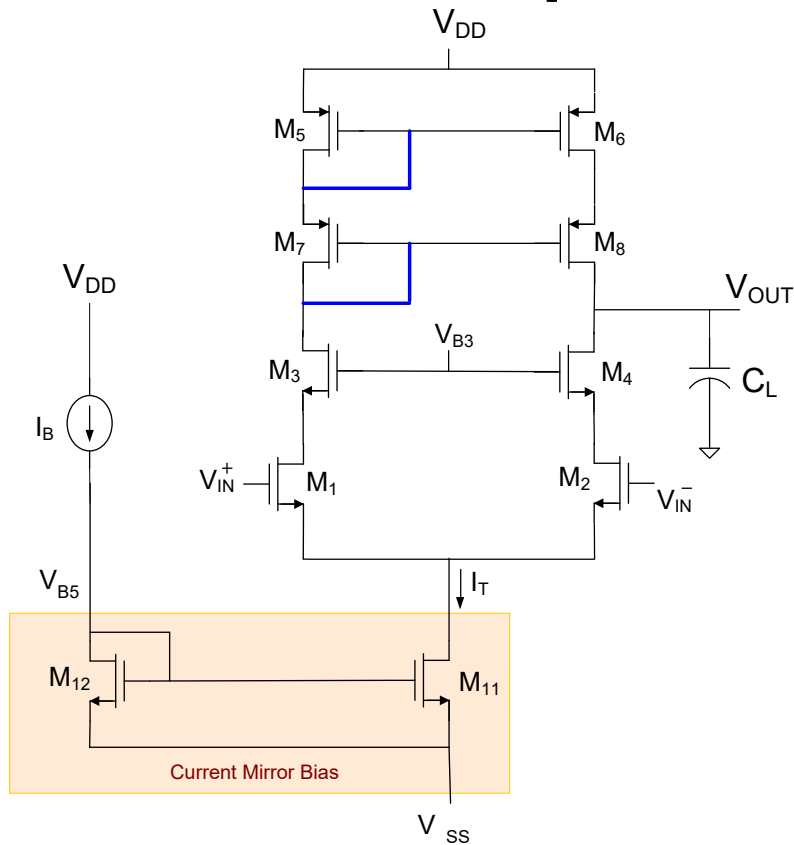
Telescopic Cascode Op Amp



- One tail bias current generator shown
- I_T often one of many outputs of a current mirror
- I_B and M_{12} often common to many blocks
- I_B often generated from a reference generator circuit
- Tail voltage bias can also be used

(CMFB circuit not shown)

Telescopic Cascode Op Amp

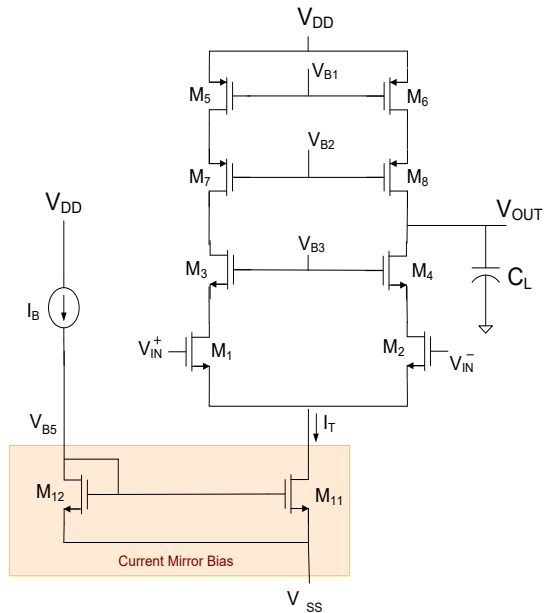


Standard p-channel Cascode Mirror

Wide-Swing p-channel Cascode Mirror

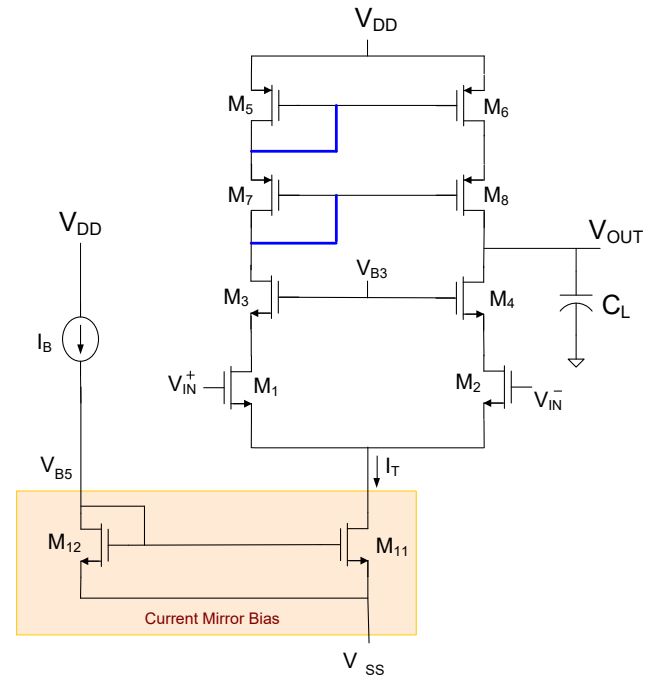
- Current-Mirror p-channel Bias to Eliminate CMFB
- Only single-ended output available
- Doubles dc gain

Telescopic Cascode Op Amp



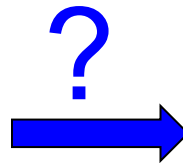
(CMFB circuit needed)

$$A_d(s) = \frac{-\frac{g_{m1}}{2}}{sC_L + g_{o1} \frac{g_{o3}}{g_{m3}} + g_{o5} \frac{g_{o7}}{g_{m7}}}$$

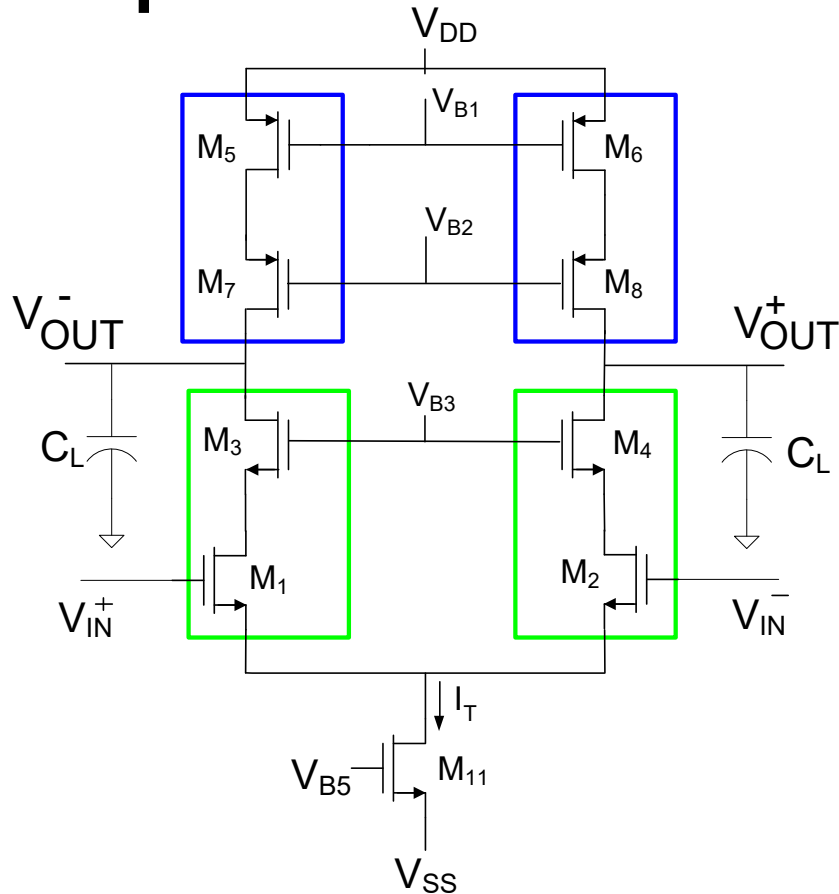


(No CMFB circuit needed)

$$A_d(s) = \frac{-g_{m1}}{sC_L + g_{o1} \frac{g_{o3}}{g_{m3}} + g_{o5} \frac{g_{o7}}{g_{m7}}}$$



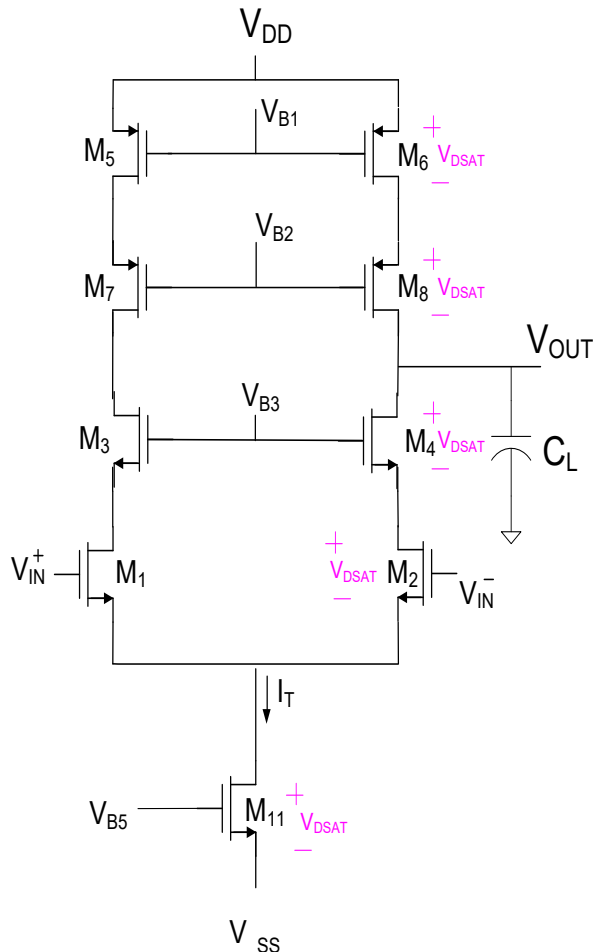
Telescopic Cascode Op Amp



- Differential Output
- CMFB to establish V_{B1} or V_{B5} needed
- Tail current generally generated with current mirror

Telescopic Cascode Op Amp

Signal Swing and Power Supply Limitations

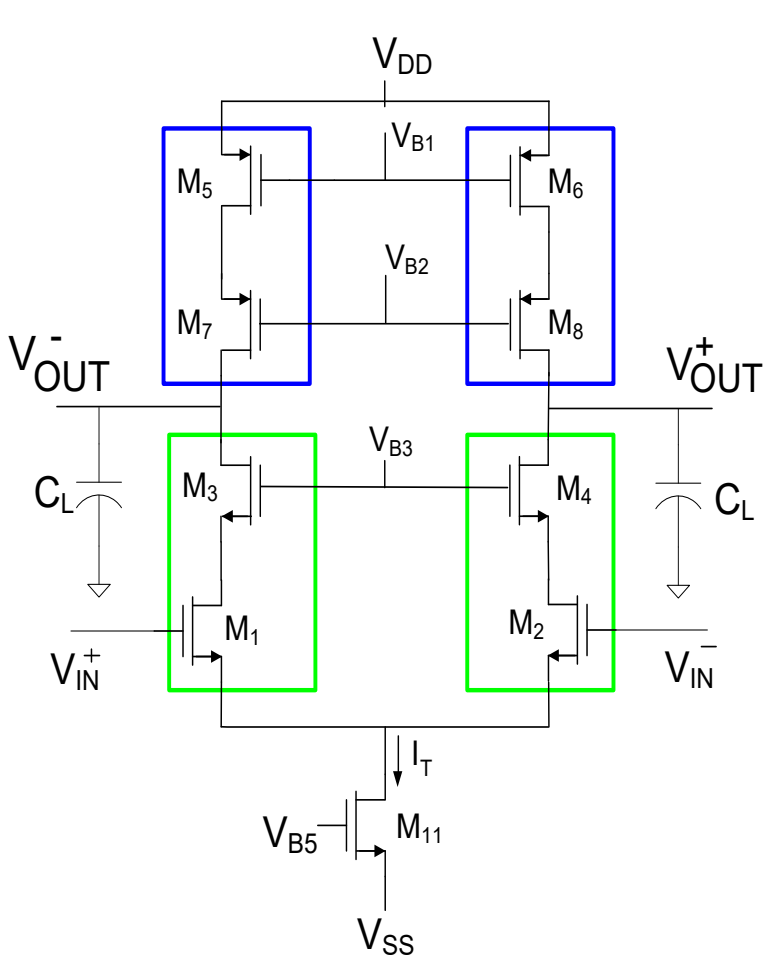


There are a minimum of $2 V_{DSAT}$ drops between V_{OUT} and V_{DD} and a minimum of $3 V_{DSAT}$ drops between V_{OUT} and V_{SS}

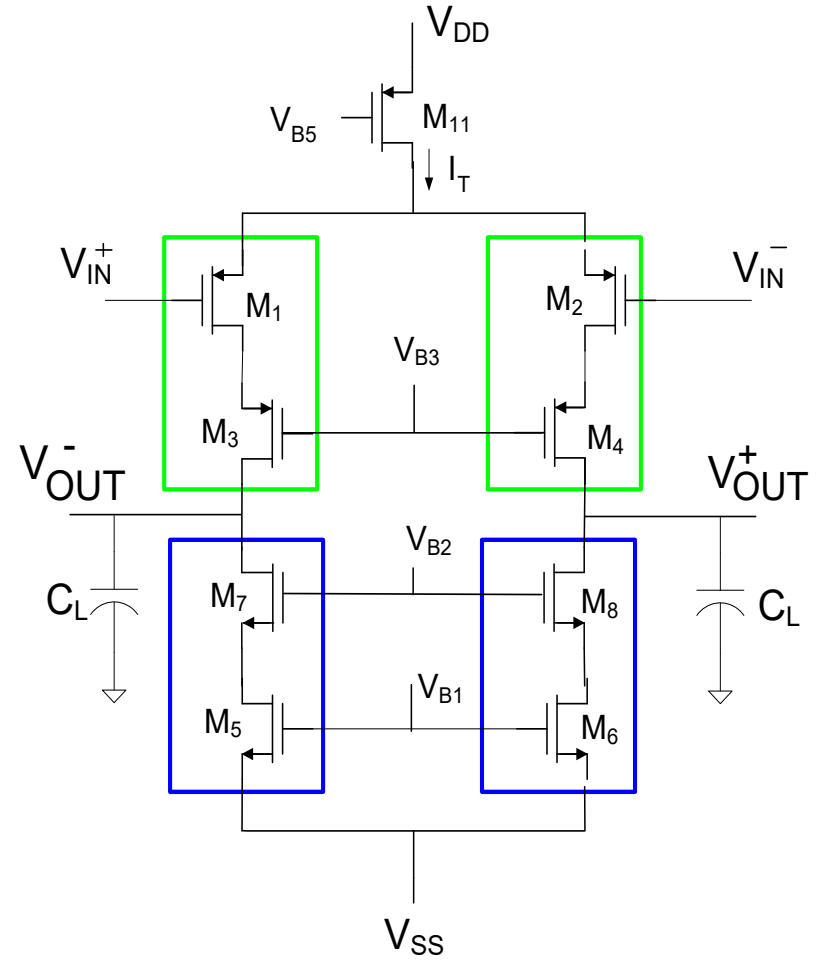
Thus, there are a minimum of $5 V_{DSAT}$ drops between V_{DD} and V_{SS}

This establishes a lower bound on $V_{DD} - V_{SS}$ and it will be further reduced by the p-p signal swing required on the output

Telescopic Cascode Op Amp



n-channel inputs

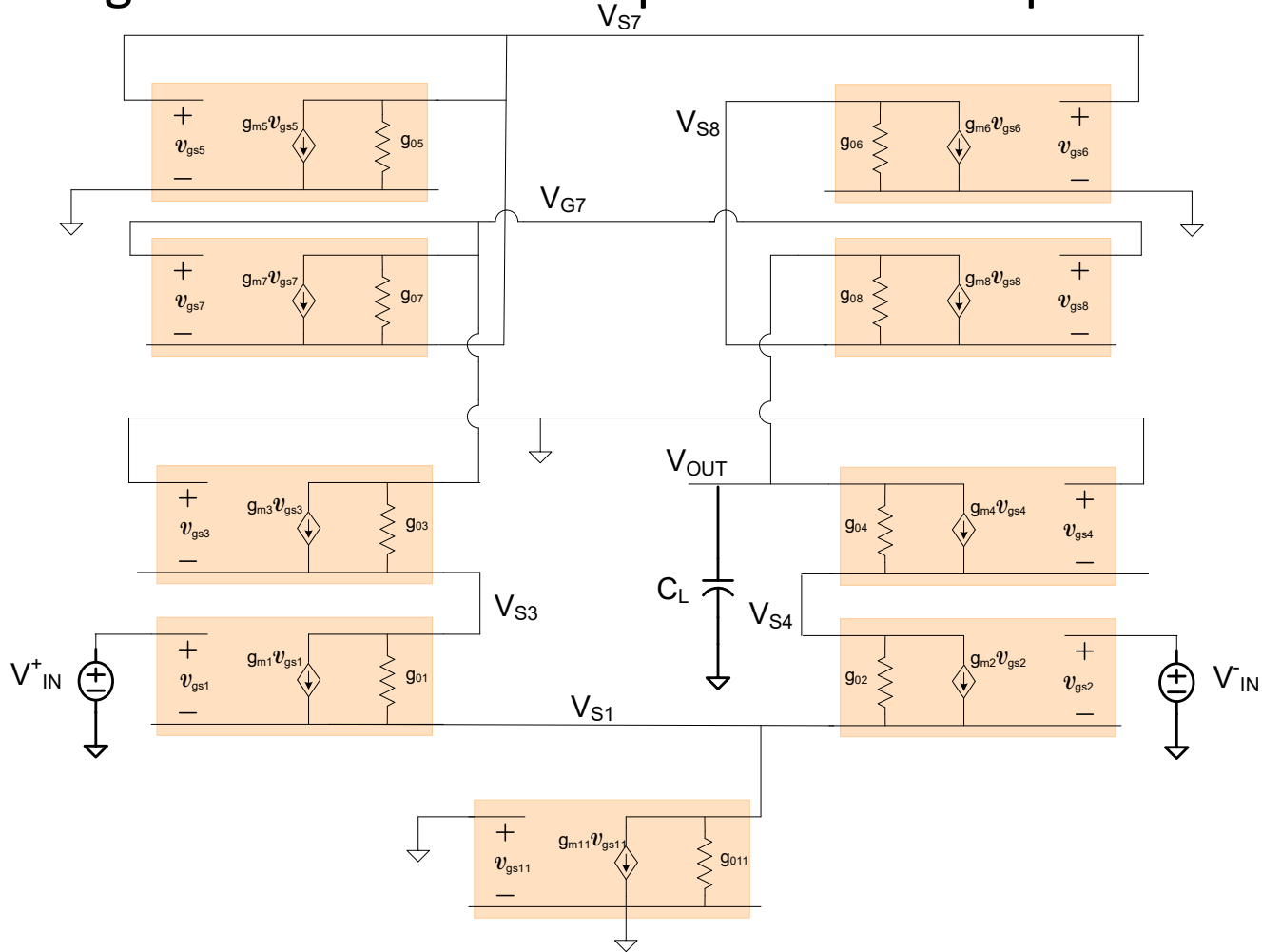


p-channel inputs

How important is it to develop good approximate analysis methods for an op amp with the complexity of the telescopic cascode structure?

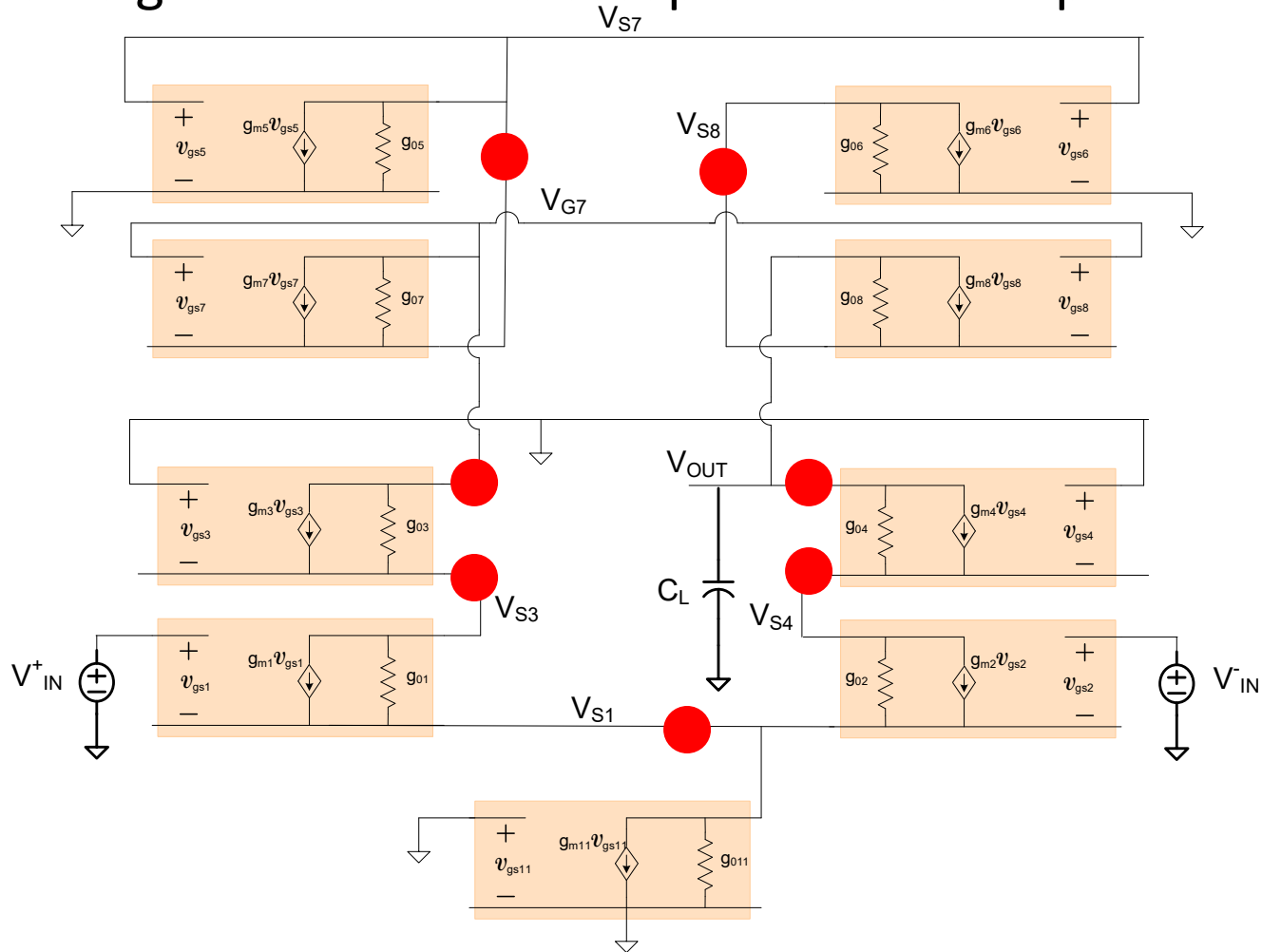
And many useful op amp circuits will have more complexity !!

Small-Signal model of Telescopic Cascode Amplifier



A bit tedious to obtain but really straight forward

Small-Signal model of Telescopic Cascode Amplifier



A bit tedious to obtain but really straight forward

● Non-ground nodes for small-signal nodal analysis

Analysis of Telescopic Cascode Amplifier

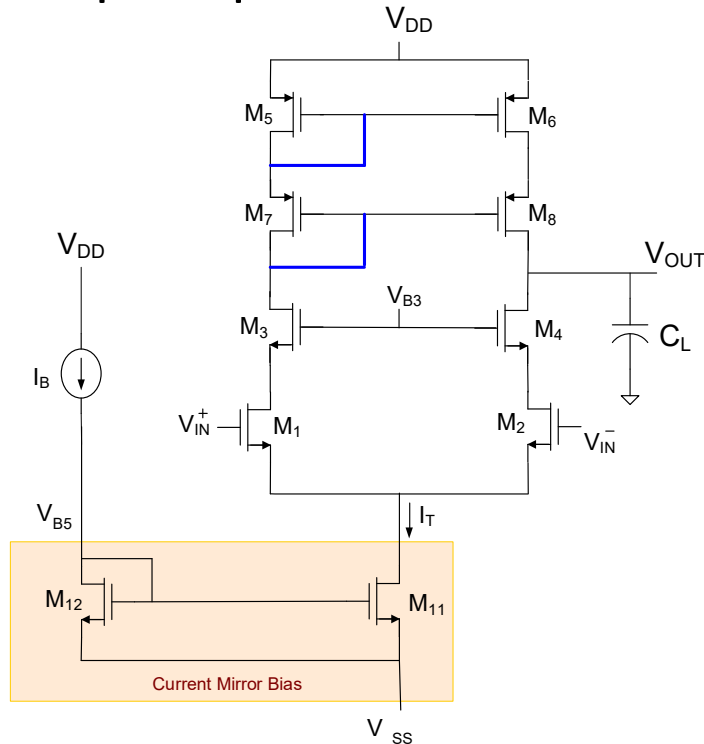
Apply KCL at 7 nodes to obtain a set of 7 independent linear equations

$$\left. \begin{aligned}
 V_{S1} (g_{01} + g_{02} + g_{011}) &= V_{S3} g_{01} + V_{S4} g_{02} + g_{m1} (V_{IN}^+ - V_{S1}) + g_{m2} (V_{IN}^- - V_{S1}) \\
 V_{S3} (g_{01} + g_{03}) + g_{m1} (V_{IN}^+ - V_{S1}) &= g_{01} V_{S1} + g_{03} V_{G7} - g_{m3} V_{S3} \\
 V_{S4} (g_{02} + g_{04}) + g_{m2} (V_{IN}^- - V_{S1}) &= g_{02} V_{S1} + g_{04} V_{OUT} - g_{m4} V_{S4} \\
 V_{OUT} (sC_L + g_{04} + g_{08}) - g_{m4} V_{S4} + g_{m8} (V_{G7} - V_{S8}) &= g_{04} V_{S4} + g_{08} V_{S8} \\
 V_{G7} (g_{07} + g_{03}) + g_{m7} (V_{G7} - V_{S7}) - g_{m3} V_{S3} &= g_{03} V_{S3} + g_{07} V_{S7} \\
 V_{S8} (g_{06} + g_{08}) + g_{m8} V_{S7} &= g_{m8} (V_{G7} - V_{S8}) + V_{OUT} g_{08} \\
 V_{S7} (g_{05} + g_{07}) + g_{m5} V_{S7} &= g_{m7} (V_{G7} - V_{S7}) + g_{07} V_{G7}
 \end{aligned} \right\}$$

A bit tedious to obtain but really straight forward

Time required to obtain set of equations is quite small

Telescopic Cascode Op Amp with Mirror-connected Counterpart Circuit



- Previous analysis obtained almost by inspection
- Gain expressions were real simple
- Gain expressions provide major insight into operation
- Some assumptions were made to simplify analysis
 - $V_{ac}=0$ at "approximate axis of symmetry"
 - Matched left and right side transistors
 - Current mirror used to mirror left-side current to right side

How much more involved would an exact analysis be ?
Would an exact analysis provide additional insight ?

Analysis of Telescopic Cascode Amplifier

Apply KCL at 7 nodes to obtain a set of 7 independent linear equations

$$\left. \begin{aligned} V_{S1}(g_{01} + g_{02} + g_{011}) &= V_{S3}g_{01} + V_{S4}g_{02} + g_{m1}(V_{IN}^+ - V_{S1}) + g_{m2}(V_{IN}^- - V_{S1}) \\ V_{S3}(g_{01} + g_{03}) + g_{m1}(V_{IN}^+ - V_{S1}) &= g_{01}V_{S1} + g_{03}V_{G7} - g_{m3}V_{S3} \\ V_{S4}(g_{02} + g_{04}) + g_{m2}(V_{IN}^- - V_{S1}) &= g_{02}V_{S1} + g_{04}V_{OUT} - g_{m4}V_{S4} \\ V_{OUT}(sC_L + g_{04} + g_{08}) - g_{m4}V_{S4} + g_{m8}(V_{G7} - V_{S8}) &= g_{04}V_{S4} + g_{08}V_{S8} \\ V_{G7}(g_{07} + g_{03}) + g_{m7}(V_{G7} - V_{S7}) - g_{m3}V_{S3} &= g_{03}V_{S3} + g_{07}V_{S7} \\ V_{S8}(g_{06} + g_{08}) + g_{m8}V_{S7} &= g_{m8}(V_{G7} - V_{S8}) + V_{OUT}g_{08} \\ V_{S7}(g_{05} + g_{07}) + g_{m5}V_{S7} &= g_{m7}(V_{G7} - V_{S7}) + g_{07}V_{G7} \end{aligned} \right\}$$

In matrix form, this looks like (0 in blank locations)

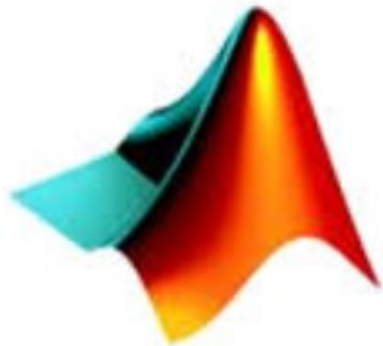
$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & & & & & & & \\ a_{21} & a_{22} & & & a_{26} & & & & & \\ a_{31} & & a_{33} & & & & a_{37} & & & \\ & & a_{43} & & a_{45} & a_{46} & a_{47} & & & \\ & a_{52} & & a_{54} & a_{56} & & & & & \\ & & a_{64} & a_{65} & a_{66} & a_{67} & & & & \\ & & a_{74} & & a_{76} & & & & & \end{bmatrix} \begin{bmatrix} V_{S1} \\ V_{S3} \\ V_{S4} \\ V_{S7} \\ V_{S8} \\ V_{G7} \\ V_{OUT} \end{bmatrix} = \begin{bmatrix} g_{m1} & g_{m2} \\ g_{m1} & 0 \\ 0 & g_{m2} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_{IN}^+ \\ V_{IN}^- \end{bmatrix}$$

$$[A] \cdot [V] = [G] \cdot [V_{IN}]$$

$$[V] = [A]^{-1} \cdot [G] \cdot [V_{IN}]$$

- The A matrix has many 0's which dramatically reduces complexity of solution
- But there are still a large number of off-diagonal terms

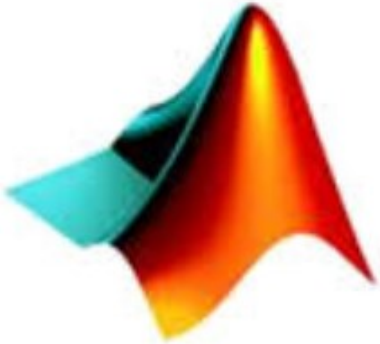
Analytical solution of these equations is not practical



MATLAB

```
%Complete Analysis of Telescopic Cascode Op Amp
diary('matlabdiary')
clear
clc
fprintf('Complete Analysis of Telescopic Cascode Op Amp')
syms Av s CL VOUT VS1 VS3 VS4 VS7 VS8 VG7 Vinn Vinp g01 g02 g03 g04 g05
g06 g07 g08 g011 gm1 gm2 gm3 gm4 gm5 gm6 gm7 gm8
eqn1 =VS1*(gm1+gm2+g01+g02+g011)==VS3*g01+VS4*g02+Vinp*gm1+gm2*Vinn;
eqn2 = VS3*(gm3+g01+g03)+gm1*Vinp==VS1*(g01+gm1)+VG7*g03;
eqn3 = VS4*(gm4+g02+g04)+Vinn*gm2==VS1*(gm2+g02)+VOUT*g04;
eqn4 = VOUT*(s*CL+g04+g08)+VG7*gm8==VS4*(gm4+g04)+VS8*(gm8+g08);
eqn5 = VG7*(gm7+g07+g03)==VS3*(gm3+g03)+VS7*(gm7+g07);
eqn6 = VS8*(gm8+g06+g08)+VS7*gm8==VG7*gm8+VOUT*g08;
eqn7 = VS7*(gm7+gm5+g05+g07)==VG7*(gm7+g07);
eqn8 = Av==VOUT/Vinp;
sol=solve([eqn1,eqn2,eqn3,eqn4,eqn5,eqn6,eqn7,eqn8],[VOUT,VS1,VS3,VS4,VS7,
VS8,VG7,Av]);
VOUTX=sol.VOUT
collect(VOUTX,Vinn)
```

Simple program and effort to write program is small



MATLAB

```
fprintf('Differential Analysis of Telescopic Cascode Op Amp')
syms Av s CL VOUT VS1 VS3 VS4 VS7 VS8 VG7 Vdiff g01 g02 g03 g04 g05 g06 g07 g08 g011 gm1 gm2 gm3
gm4 gm5 gm6 gm7 gm8
eqn1 = VS1*(gm1+gm2+g01+g02+g011)==VS3*g01+VS4*g02+Vdiff*gm1/2-gm2*Vdiff/2;
eqn2 = VS3*(gm3+g01+g03)+gm1*Vdiff/2==VS1*(g01+gm1)+VG7*g03;
eqn3 = VS4*(gm4+g02+g04)-Vdiff*gm2/2==VS1*(gm2+g02)+VOUT*g04;
eqn4 = VOUT*(s*CL+g04+g08)+VG7*gm8==VS4*(gm4+g04)+VS8*(gm8+g08);
eqn5 = VG7*(gm7+g07+g03)==VS3*(gm3+g03)+VS7*(gm7+g07);
eqn6 = VS8*(gm8+g06+g08)+VS7*gm8==VG7*gm8+VOUT*g08;
eqn7 = VS7*(gm7+gm5+g05+g07)==VG7*(gm7+g07);
eqn8 = Av==VOUT/Vdiff;
sol=solve([eqn1,eqn2,eqn3,eqn4,eqn5,eqn6,eqn7,eqn8],[VOUT,VS1,VS3,VS4,VS7,VS8,VG7,Av]);
VOUTX=sol.VOUT;
GainAv=sol.Av
collect(GainAv,s)
```

Simple program and effort to write program is small

MATLAB Solution Continued 2:

```
g01*g03*g05*g06*g08*gm4*gm7 + g01*g03*g06*g07*g08*gm4*gm5 + g01*g04*g05*g06*g08*gm3*gm7 +
g01*g04*g06*g07*g08*gm3*gm5 + g01*g05*g06*g07*g08*gm3*gm4 + g03*g04*g05*g06*g08*gm1*gm7 +
g03*g04*g06*g07*g08*gm1*gm5 + g03*g05*g06*g07*g08*gm1*gm4 + g04*g05*g06*g07*g08*gm1*gm3 +
g02*g03*g04*g06*g08*gm5*gm7 + g02*g03*g05*g06*g08*gm4*gm7 + g02*g03*g06*g07*g08*gm4*gm5 +
g02*g04*g05*g06*g08*gm3*gm7 + g02*g04*g06*g07*g08*gm3*gm5 + g02*g05*g06*g07*g08*gm3*gm4 +
g03*g04*g05*g06*g08*gm2*gm7 + g03*g04*g06*g07*g08*gm2*gm5 + g03*g05*g06*g07*g08*gm2*gm4 +
g04*g05*g06*g07*g08*gm2*gm3 + g01*g02*g03*g04*g011*gm7*gm8 + g01*g02*g04*g06*g011*gm5*gm7 +
g01*g02*g04*g05*g011*gm7*gm8 + g01*g02*g04*g07*g011*gm5*gm8 + g01*g02*g04*g08*g011*gm5*gm7 +
g01*g03*g06*g08*gm1*gm4*gm5 + g02*g03*g04*g06*g011*gm5*gm7 + g02*g04*g05*g06*g011*gm3*gm7 +
g02*g04*g06*g07*g011*gm3*gm5 + g01*g02*g06*g08*gm1*gm5*gm7 + g01*g03*g06*g08*gm1*gm4*gm7 +
g02*g03*g04*g05*g011*gm7*gm8 + g02*g03*g04*g07*g011*gm5*gm8 + g02*g03*g04*g08*gm1*gm5*gm7 +
g02*g04*g05*g07*g011*gm3*gm8 + g02*g04*g05*g08*gm1*gm3*gm7 + g02*g04*g07*g08*gm1*gm3*gm5 +
g02*g04*g06*g08*gm1*gm5*gm7 + g02*g04*g07*g08*gm1*gm3*gm5 + g02*g04*g08*gm1*gm3*gm5 +
g01*g04*g06*g08*gm1*gm5*gm7 + g01*g05*g06*g08*gm1*gm4*gm7 + g01*g06*g07*g08*gm1*gm3*gm5 +
g02*g03*g06*g08*gm1*gm3*gm7 + g02*g05*g06*g08*gm1*gm3*gm5 + g02*g06*g07*g08*gm1*gm3*gm5 +
g02*g04*g06*g08*gm1*gm3*gm5 + g02*g05*g06*g08*gm1*gm3*gm5 + g02*g06*g07*g08*gm1*gm3*gm5 +
g01*g02*g06*g08*gm4*gm5*gm7 + g01*g04*g06*g08*gm2*gm5*gm7 + g01*g05*g06*g08*gm2*gm4*gm7 +
g01*g06*g07*g08*gm2*gm4*gm5 + g02*g03*g04*g08*gm1*gm7*gm8 + g02*g04*g07*g08*gm1*gm3*gm8 +
g01*g03*g06*g08*gm4*gm5*gm7 + g01*g04*g06*g08*gm3*gm5*gm7 + g01*g05*g06*g08*gm3*gm4*gm7 +
g03*g06*g07*g08*gm1*gm4*gm5 + g04*g05*g06*g08*gm1*gm3*gm7 + g04*g06*g07*g08*gm1*gm3*gm5 +
g05*g06*g07*g08*gm1*gm3*gm4 + g02*g03*g06*g08*gm4*gm5*gm7 + g02*g04*g06*g08*gm3*gm5*gm7 +
g02*g05*g06*g08*gm3*gm4*gm7 + g02*g06*g07*g08*gm3*gm4*gm5 + g03*g04*g06*g08*gm2*gm5*gm7 +
g03*g05*g06*g08*gm2*gm4*gm7 + g03*g06*g07*g08*gm2*gm4*gm5 + g05*g06*g07*g08*gm2*gm3*gm4 +
g04*g06*g07*g08*gm2*gm3*gm5 + g02*g04*g06*g011*gm3*gm5*gm8 + g02*g04*g07*g011*gm3*gm5*gm8 +
g02*g06*g08*gm1*gm3*gm5*gm7 + g03*g06*g08*gm1*gm4*gm5*gm7 + g04*g06*g08*gm1*gm3*gm5*gm7 +
g05*g06*g08*gm1*gm3*gm4*gm7 + g02*g04*g06*gm1*gm3*gm5*gm8 + g02*g04*g07*gm1*gm3*gm5*gm8 +
g02*g04*g08*gm1*gm3*gm5*gm7 + g02*g04*g06*gm1*gm3*gm7*gm8 + g02*g04*g08*gm1*gm3*gm5*gm7 +
g02*g04*g06*gm1*gm3*gm7*gm8 + g02*g06*g08*gm1*gm3*gm5*gm7 + g01*g06*g08*gm3*gm4*gm5*gm7 +
g04*g06*g08*gm1*gm3*gm5*gm7 + g02*g06*g08*gm2*gm4*gm5*gm7 + g04*g06*g08*gm2*gm3*gm5*gm7 +
g05*g06*g08*gm2*gm3*gm4*gm7 + g06*g07*g08*gm2*gm3*gm4*gm5 + g02*g04*g011*gm3*gm5*gm7*gm8 +
g06*g08*gm1*gm3*gm4*gm5*gm7 + g06*g08*gm2*gm3*gm4*gm5*gm7 + CL*g01*g02*g03*g04*g05*g06*s +
CL*g01*g02*g03*g04*g06*g07*s + CL*g01*g02*g03*g05*g06*g07*s + CL*g01*g02*g03*g04*g07*g08*s +
CL*g01*g02*g04*g05*g06*g07*s + CL*g01*g02*g03*g04*g05*g07*g08*s + CL*g01*g02*g03*g04*g05*g06*g07*s +
CL*g01*g02*g03*g04*g05*g07*g08*s + CL*g01*g02*g03*g04*g05*g06*g011*s + CL*g01*g02*g03*g05*g08*gm1*s +
CL*g01*g02*g03*g04*g05*g07*gm1*s + CL*g01*g03*g04*g05*g08*gm1*s + CL*g01*g03*g04*g06*g07*gm1*s +
CL*g02*g03*g05*g06*g07*gm1*s + CL*g01*g04*g05*g07*gm1*s + CL*g02*g03*g05*g07*gm1*s + CL*g03*g04*g05*g06*gm5*s +
CL*g01*g02*g03*g05*g06*gm4*s + CL*g01*g03*g04*g05*g06*gm2*s + CL*g01*g02*g03*g04*g05*gm8*s +
CL*g01*g02*g03*g04*g06*gm7*s + CL*g01*g02*g03*g04*g08*gm5*s + CL*g01*g02*g03*g05*g08*gm4*s +
CL*g01*g02*g03*g06*g07*gm2*s + CL*g01*g03*g04*g05*g08*gm2*s + CL*g01*g03*g04*g06*g07*gm2*s +
CL*g01*g02*g03*g05*g06*gm7*s + CL*g01*g02*g03*g06*g07*gm3*s + CL*g01*g02*g05*g06*gm7*gm3*s +
CL*g02*g03*g05*g06*gm7*gm3*s + CL*g01*g02*g03*g04*g07*gm3*s + CL*g01*g02*g03*g04*g08*gm7*s +
CL*g01*g02*g03*g05*g06*gm7*s + CL*g02*g03*g04*g06*g07*gm5*s + CL*g01*g02*g04*g06*g07*gm5*s +
CL*g01*g02*g05*g06*g07*gm4*s + CL*g01*g03*ng04*g07*gm2*s + CL*g01*g04*g05*g06*g07*gm2*s +
CL*g01*g02*g03*g05*g07*gm8*s + CL*g01*g02*g03*g05*g08*gm7*s + CL*g01*g02*g03*g07*gm8*gm5*s +
CL*g01*g02*g05*g07*gm8*gm3*s + CL*g01*g03*g04*g05*g06*gm7*s + CL*g01*g04*g05*g06*g07*gm3*s +
CL*g02*g03*g04*g05*g06*g07*gm1*s + CL*g01*g02*g04*g05*g07*gm8*s + CL*g01*g02*g04*g05*g08*gm7*s +
CL*g01*g02*g04*g07*gm8*gm5*s + CL*g01*g02*g05*g07*gm8*gm4*s + CL*g01*g04*g05*g07*gm8*gm2*s +
CL*g02*g03*g04*g05*g06*gm7*s + CL*g02*g03*g04*g06*g07*gm5*s + CL*g02*g03*g05*g06*g07*gm4*s +
CL*g02*g04*g05*g06*g07*gm3*s + CL*g03*g04*g05*g06*g07*gm2*s + CL*g01*g02*g03*g04*g06*gm5*s +
CL*g01*g03*g04*g05*g08*gm7*s + CL*g01*g03*g04*g07*gm8*gm5*s +
```


MATLAB Solution Continued 4:

```
CL*g01*g03*g07*gm2*gm4*gm8*s + CL*g01*g03*g08*gm2*gm4*gm7*s + CL*g01*g04*g06*gm2*gm5*gm7*s +
CL*g01*g05*g06*gm2*gm4*gm7*s + CL*g01*g06*g07*gm2*gm4*gm5*s + CL*g01*g02*g03*gm5*gm7*gm8*s +
CL*g01*g02*g05*gm3*gm7*gm8*s + CL*g01*g02*g07*gm3*gm5*gm8*s + CL*g01*g02*g08*gm3*gm5*gm7*s +
CL*g01*g03*g06*gm4*gm5*gm7*s + CL*g01*g04*g06*gm3*gm5*gm7*s + CL*g01*g05*g06*gm3*gm4*gm7*s +
CL*g01*g06*g07*gm3*gm4*gm5*s + CL*g02*g03*g05*gm1*gm7*gm8*s + CL*g02*g03*g07*gm1*gm5*gm8*s +
CL*g02*g03*g08*gm1*gm5*gm7*s + CL*g02*g05*g07*gm1*gm3*gm8*s + CL*g02*g05*g08*gm1*gm3*gm7*s +
CL*g02*g07*g08*gm1*gm3*gm5*s + CL*g03*g04*g06*gm1*gm5*gm7*s + CL*g03*g05*g06*gm1*gm4*gm7*s +
CL*g03*g06*g07*gm1*gm4*gm5*s + CL*g04*g05*g06*gm1*gm3*gm7*s + CL*g04*g06*g07*gm1*gm3*gm5*s +
CL*g05*g06*g07*gm1*gm3*gm4*s + CL*g01*g02*g04*gm5*gm7*gm8*s + CL*g01*g02*g05*gm4*gm7*gm8*s +
CL*g01*g02*g07*gm4*gm5*gm8*s + CL*g01*g02*g08*gm4*gm5*gm7*s + CL*g01*g04*g05*gm2*gm7*gm8*s +
CL*g01*g04*g07*gm2*gm5*gm8*s + CL*g01*g05*g07*gm2*gm4*gm8*s + CL*g01*g05*g08*gm2*gm4*gm8*s +
CL*g01*g05*g08*gm2*gm4*gm7*s + CL*g01*g07*g08*gm2*gm4*gm5*s + CL*g02*g03*g06*gm4*gm5*gm7*s +
CL*g02*g04*g06*gm3*gm5*gm7*s + CL*g02*g05*g06*gm3*gm4*gm7*s + CL*g02*g06*g07*gm3*gm4*gm5*s +
CL*g03*g04*g06*gm2*gm5*gm7*s + CL*g03*g05*g06*gm2*gm4*gm7*s + CL*g03*g06*g07*gm2*gm4*gm5*s +
CL*g04*g05*g06*gm2*gm3*gm7*s + CL*g04*g06*g07*gm2*gm3*gm5*s + CL*g05*g06*g07*gm2*gm3*gm4*s +
CL*g01*g03*g04*gm5*gm7*gm8*s + CL*g01*g03*g05*gm4*gm7*gm8*s + CL*g01*g03*g07*gm4*gm5*gm8*s +
CL*g01*g03*g08*gm4*gm5*gm7*s + CL*g01*g04*g05*gm3*gm7*gm8*s + CL*g01*g04*g07*gm3*gm5*gm8*s +
CL*g01*g04*g08*gm3*gm5*gm7*s + CL*g01*g07*g08*gm3*gm4*gm5*s + CL*g03*g04*g05*gm1*gm5*gm8*s +
CL*g03*g04*g07*gm1*gm4*gm8*s + CL*g03*g05*g07*gm1*gm4*gm8*s + CL*g03*g05*g08*gm1*gm4*gm7*s +
CL*g03*g07*g08*gm1*gm4*gm5*s + CL*g04*g05*g07*gm1*gm3*gm8*s + CL*g04*g05*g08*gm1*gm3*gm7*s +
CL*g04*g07*gm1*gm3*gm5*s + CL*g02*g03*g04*gm5*gm7*gm8*s + CL*g02*g03*g04*gm5*gm7*gm8*s +
CL*g02*g03*g08*gm4*gm5*gm7*s + CL*g02*g04*g05*gm3*gm7*gm8*s + CL*g02*g04*g08*gm3*gm5*gm7*s +
CL*g02*g05*g07*gm3*gm4*gm8*s + CL*g02*g05*g08*gm3*gm4*gm7*s + CL*g02*g07*g08*gm3*gm4*gm5*s +
CL*g03*g04*g05*gm2*gm7*gm8*s + CL*g03*g04*g07*gm2*gm5*gm8*s + CL*g03*g04*g08*gm2*gm5*gm7*s +
CL*g03*g05*g07*gm2*gm4*gm8*s + CL*g03*g05*g08*gm2*gm4*gm7*s + CL*g03*g07*g08*gm2*gm4*gm5*s +
CL*g05*g07*g08*gm2*gm3*gm4*s + CL*g01*g03*g011*gm4*gm5*gm8*s + CL*g01*g02*g011*gm5*gm7*gm8*s +
CL*g01*g04*g011*gm5*gm7*gm8*s + CL*g01*g05*g011*gm4*gm7*gm8*s + CL*g01*g07*gm11*gm4*gm5*gm8*s +
CL*g01*g08*gm11*gm4*gm5*gm7*s + CL*g02*g03*g011*gm5*gm7*gm8*s + CL*g02*g05*g011*gm3*gm5*gm7*gm8*s +
CL*g02*g07*g011*gm3*gm5*gm8*s + CL*g02*g08*gm11*gm3*gm5*gm7*s + CL*g03*g06*g011*gm4*gm5*gm7*s +
CL*g03*g06*g07*gm11*gm3*gm5*gm7*s + CL*g03*g06*g07*gm11*gm3*gm4*gm5*s + CL*g06*g07*gm11*gm3*gm4*gm5*s +
CL*g03*g04*g011*gm5*gm7*gm8*s + CL*g03*g05*g011*gm4*gm7*gm8*s + CL*g03*g07*gm11*gm4*gm5*gm8*s +
CL*g03*g08*gm11*gm4*gm5*gm7*s + CL*g04*g05*g011*gm3*gm7*gm8*s + CL*g04*g07*gm11*gm3*gm5*gm8*s +
CL*g04*g08*gm11*gm3*gm5*gm7*s + CL*g05*g07*gm11*gm3*gm4*gm8*s + CL*g05*g08*gm11*gm3*gm4*gm7*s +
CL*g07*g08*gm11*gm3*gm4*gm5*s + CL*g01*g03*gm2*gm4*gm5*gm8*s + CL*g02*g06*gm1*gm3*gm5*gm7*s +
CL*g01*g03*gm2*gm4*gm7*gm8*s + CL*g01*g06*gm2*gm4*gm5*gm7*s + CL*g01*g02*gm3*gm5*gm7*gm8*s +
CL*g01*g06*gm3*gm4*gm5*gm7*s + CL*g02*g03*gm1*gm5*gm7*gm8*s + CL*g02*g05*gm1*gm3*gm7*gm8*s +
CL*g02*g07*gm1*gm3*gm5*gm8*s + CL*g02*g08*gm1*gm3*gm5*gm7*s + CL*g03*g06*gm1*gm4*gm5*gm7*s +
CL*g04*g06*gm1*gm3*gm5*gm7*s + CL*g05*g06*gm1*gm3*gm4*gm7*s + CL*g06*g07*gm1*gm3*gm4*gm5*s +
CL*g01*g02*gm4*gm5*gm8*s + CL*g01*g04*gm2*gm5*gm7*gm8*s + CL*g01*g05*gm2*gm4*gm7*gm8*s +
CL*g01*g07*gm2*gm4*gm5*gm8*s + CL*g01*g08*gm2*gm4*gm5*gm7*s + CL*g02*g06*gm3*gm4*gm5*gm7*s +
CL*g03*g06*gm2*gm4*gm5*gm7*s + CL*g04*g06*gm2*gm3*gm5*gm7*s + CL*g05*g06*gm2*gm3*gm4*gm7*s +
CL*g06*g07*gm2*gm3*gm4*gm5*s + CL*g01*g03*gm4*gm5*gm7*gm8*s + CL*g01*g04*gm3*gm5*gm7*gm8*s +
CL*g01*g05*gm3*gm4*gm7*gm8*s + CL*g03*g05*gm1*gm4*gm7*gm8*s + CL*g03*g07*gm1*gm4*gm5*gm8*s +
CL*g03*g08*gm1*gm4*gm5*gm7*s + CL*g04*g05*gm1*gm3*gm7*gm8*s + CL*g04*g07*gm1*gm3*gm5*gm8*s +
CL*g04*g08*gm1*gm3*gm4*gm7*s + CL*g05*g07*gm1*gm3*gm4*gm8*s + CL*g05*g08*gm1*gm3*gm4*gm7*s +
CL*g07*g08*gm1*gm3*gm4*gm5*s + CL*g02*g03*gm4*gm5*gm7*gm8*s + CL*g02*g04*gm3*gm5*gm7*gm8*s +
CL*g02*g05*gm3*gm4*gm7*gm8*s + CL*g02*g07*gm3*gm4*gm5*gm8*s + CL*g02*g08*gm3*gm4*gm5*gm7*s +
CL*g03*g04*gm2*gm5*gm7*gm8*s + CL*g03*g05*gm2*gm4*gm7*gm8*s + CL*g03*g07*gm2*gm4*gm5*gm8*s +
CL*g03*g08*gm2*gm4*gm5*gm7*s + CL*g04*g05*gm2*gm3*gm7*gm8*s + CL*g04*g07*gm2*gm3*gm5*gm8*s +
CL*g04*g08*gm2*gm3*gm5*gm7*s + CL*g05*g07*gm2*gm3*gm4*gm7*s + CL*g05*g08*gm2*gm3*gm4*gm7*s +
CL*g07*g08*gm2*gm3*gm4*gm5*s + CL*g01*g011*gm3*gm4*gm5*gm7*s + CL*g04*g011*gm3*gm5*gm7*gm8*s +
CL*g05*g011*gm3*gm4*gm7*gm8*s + CL*g07*g011*gm3*gm4*gm5*gm8*s + CL*g08*gm11*gm3*gm4*gm5*gm7*s +
CL*g02*gm1*gm3*gm5*gm7*gm8*s + CL*g06*gm1*gm3*gm4*gm5*gm7*s + CL*g01*gm2*gm4*gm5*gm7*gm8*s +
CL*g06*gm2*gm3*gm4*gm5*gm7*s + CL*g01*gm3*gm4*gm5*gm7*gm8*s + CL*g03*gm1*gm4*gm5*gm7*gm8*s +
CL*g04*gm1*gm3*gm5*gm7*gm8*s + CL*g05*gm1*gm3*gm4*gm7*gm8*s + CL*g07*gm1*gm3*gm4*gm5*gm8*s +
CL*g08*gm1*gm3*gm4*gm5*gm7*s + CL*g02*gm3*gm4*gm5*gm7*gm8*s + CL*g03*gm2*gm4*gm5*gm7*gm8*s +
CL*g04*gm2*gm3*gm4*gm5*gm7*gm8*s + CL*g05*gm2*gm3*gm4*gm7*gm8*s + CL*g07*gm2*gm3*gm4*gm5*gm8*s +
CL*g08*gm2*gm3*gm4*gm5*gm7*gm8*s + CL*gm1*gm3*gm4*gm5*gm7*gm8*s + CL*gm1*gm3*gm4*gm5*gm7*gm8*s +
CL*gm2*gm3*gm4*gm5*gm7*gm8*s)) Vinn + (Vinp*g02*g03*g04*g07*gm1*gm8^2 +
Vinp*g02*g03*g07*gm1*gm8^2 + Vinp*g03*g04*g07*gm1*gm8^2 + Vinp*g02*g03*g04*g07*gm1*gm8^2 +
Vinp*g02*g03*g04*gm1*gm7*gm8^2 + Vinp*g02*g03*g07*gm1*gm4*gm8^2 + Vinp*g02*g04*g07*gm1*gm3*gm8^2 +
Vinp*g03*g04*g07*gm1*gm2*gm8^2 + Vinp*g02*g03*g011*gm1*gm7*gm8^2 +
Vinp*g02*g07*gm11*gm1*gm3*gm8^2 + Vinp*g03*g04*gm11*gm1*gm7*gm8^2 +
Vinp*g03*g07*gm11*gm1*gm4*gm8^2 + Vinp*g04*g07*gm11*gm1*gm3*gm8^2 +
Vinp*g02*g03*gm1*gm4*gm7*gm8^2 + Vinp*g02*g04*gm1*gm3*gm7*gm8^2 + Vinp*g02*g07*gm1*gm3*gm4*gm8^2
```


MATLAB Solution Continued 5:

```
+ Vinp*g03*g04*gm1*gm2*gm7*gm8^2 + Vinp*g03*g07*gm1*gm2*gm4*gm8^2 +
Vinp*g04*g07*gm1*gm2*gm3*gm8^2 + Vinp*g02*g011*gm1*gm3*gm7*gm8^2 +
Vinp*g03*g011*gm1*gm4*gm7*gm8^2 + Vinp*g04*g011*gm1*gm3*gm7*gm8^2 +
Vinp*g07*g011*gm1*gm3*gm4*gm8^2 + Vinp*g02*gm1*gm3*gm4*gm7*gm8^2 +
Vinp*g03*gm1*gm2*gm4*gm7*gm8^2 + Vinp*g04*gm1*gm2*gm3*gm7*gm8^2 + Vinp*g07*gm1*gm2*gm3*gm4*gm8^2
+ Vinp*g011*gm1*gm3*gm4*gm7*gm8^2 + Vinp*gm1*gm2*gm3*gm4*gm7*gm8^2 +
Vinp*g02*g03*g04*g05*g06*g07*gm1 + Vinp*g02*g03*g04*g05*g07*g08*gm1 +
Vinp*g02*g03*g04*g05*g06*gm1*gm7 + Vinp*g02*g03*g04*g06*g07*gm1*gm5 +
Vinp*g02*g03*g05*g06*g07*gm1*gm4 + Vinp*g02*g04*g05*g06*g07*gm1*gm3 +
Vinp*g03*g04*g05*g06*g07*gm1*gm2 + Vinp*g02*g03*g04*g05*g06*gm1*gm8 +
Vinp*g02*g03*g04*g05*g07*gm1*gm8 + Vinp*g02*g03*g04*g05*g08*gm1*gm7 +
Vinp*g02*g03*g04*g07*g08*gm1*gm5 + Vinp*g02*g03*g05*g07*g08*gm1*gm4 +
Vinp*g02*g04*g05*g07*g08*gm1*gm3 + Vinp*g03*g04*g05*g07*g08*gm1*gm2 +
Vinp*g02*g03*g04*g06*g07*gm1*gm8 + Vinp*g02*g03*g04*g07*g08*gm1*gm8 +
Vinp*g02*g03*g05*g06*g011*gm1*gm8 + Vinp*g02*g03*g06*g07*g011*gm1*gm8 +
Vinp*g03*g04*g05*g06*g011*gm1*gm8 + Vinp*g02*g03*g07*g08*g011*gm1*gm8 +
Vinp*g03*g04*g06*g07*g011*gm1*gm8 + Vinp*g03*g04*g07*g08*g011*gm1*gm8 +
Vinp*g02*g03*g04*g06*gm1*gm5*gm7 + Vinp*g02*g03*g05*g06*gm1*gm4*gm7 +
Vinp*g02*g03*g06*g07*gm1*gm4*gm5 + Vinp*g02*g04*g05*g06*gm1*gm3*gm7 +
Vinp*g02*g04*g06*g07*gm1*gm3*gm5 + Vinp*g03*g04*g06*g07*gm1*gm2*gm5 +
Vinp*g03*g05*g06*g07*gm1*gm2*gm4 + Vinp*g04*g05*g06*g07*gm1*gm2*gm3 +
Vinp*g02*g03*g04*g06*gm1*gm5*gm8 + Vinp*g02*g03*g05*g06*gm1*gm4*gm8 +
Vinp*g02*g04*g05*g06*gm1*gm3*gm8 + Vinp*g03*g04*g05*g06*gm1*gm2*gm8 +
Vinp*g02*g03*g04*g05*gm1*gm7*gm8 + Vinp*g02*g03*g04*g07*gm1*gm5*gm8 +
Vinp*g02*g03*g04*g08*gm1*gm5*gm7 + Vinp*g02*g03*g05*g07*gm1*gm4*gm8 +
Vinp*g02*g03*g05*g08*gm1*gm4*gm7 + Vinp*g02*g03*g07*gm1*gm3*gm8 +
Vinp*g02*g04*g05*g07*gm1*gm3*gm5 + Vinp*g02*g04*g05*g08*gm1*gm3*gm7 +
Vinp*g02*g04*g07*g08*gm1*gm3*gm5 + Vinp*g02*g05*g07*g08*gm1*gm3*gm4 +
Vinp*g03*g04*g05*g07*gm1*gm2*gm8 + Vinp*g03*g04*g05*g08*gm1*gm2*gm7 +
Vinp*g03*g04*g07*g08*gm1*gm2*gm5 + Vinp*g03*g05*g07*g08*gm1*gm2*gm4 +
Vinp*g04*g05*g07*g08*gm1*gm2*gm3 + Vinp*g02*g03*g04*g06*gm1*gm7*gm8 +
Vinp*g02*g03*g06*g07*gm1*gm4*gm8 + Vinp*g02*g04*g06*g07*gm1*gm3*gm8 +
Vinp*g03*g04*g06*g07*gm1*gm2*gm8 + Vinp*g02*g03*g06*g011*gm1*gm5*gm8 +
Vinp*g03*g05*g06*g011*gm1*gm4*gm8 + Vinp*g04*g05*g06*g011*gm1*gm3*gm8 +
Vinp*g02*g03*g08*g011*gm1*gm7*gm8 + Vinp*g02*g07*g08*g011*gm1*gm3*gm8 +
Vinp*g03*g04*g06*g011*gm1*gm7*gm8 + Vinp*g03*g06*g07*g011*gm1*gm4*gm8 +
Vinp*g04*g06*g07*g011*gm1*gm3*gm8 + Vinp*g03*g04*g08*g011*gm1*gm7*gm8 +
Vinp*g03*g05*g06*g011*gm1*gm3*gm5*gm8 + Vinp*g03*g06*g011*gm1*gm3*gm5*gm8 +
Vinp*g05*g06*g011*gm1*gm3*gm4*gm8 + Vinp*g02*g08*g011*gm1*gm3*gm7*gm8 +
Vinp*g03*g06*g011*gm1*gm4*gm7*gm8 + Vinp*g04*g06*g011*gm1*gm3*gm7*gm8 +
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MATLAB Solution Continued 6:

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Vinp*g06*g07*g011*gm1*gm3*gm4*gm8 + Vinp*g03*g08*g011*gm1*gm4*gm7*gm8 +  
Vinp*g04*g08*g011*gm1*gm3*gm7*gm8 + Vinp*g07*g08*g011*gm1*gm3*gm4*gm8 +  
Vinp*g02*g06*gm1*gm3*gm4*gm5*gm7 + Vinp*g03*g06*gm1*gm2*gm4*gm5*gm7 +  
Vinp*g04*g06*gm1*gm2*gm3*gm5*gm7 + Vinp*g05*g06*gm1*gm2*gm3*gm4*gm7 +  
Vinp*g06*g07*gm1*gm2*gm3*gm4*gm5 + Vinp*g02*g06*gm1*gm3*gm4*gm5*gm8 +  
Vinp*g03*g06*gm1*gm2*gm4*gm5*gm8 + Vinp*g04*g06*gm1*gm2*gm3*gm5*gm8 +  
Vinp*g05*g06*gm1*gm2*gm3*gm4*gm8 + Vinp*g02*g03*gm1*gm4*gm5*gm7*gm8 +  
Vinp*g02*g04*gm1*gm3*gm5*gm7*gm8 + Vinp*g02*g05*gm1*gm3*gm4*gm7*gm8 +  
Vinp*g02*g07*gm1*gm3*gm4*gm5*gm8 + Vinp*g02*g08*gm1*gm3*gm4*gm5*gm7 +  
Vinp*g03*g04*gm1*gm2*gm5*gm7*gm8 + Vinp*g03*g05*gm1*gm2*gm4*gm7*gm8 +  
Vinp*g03*g07*gm1*gm2*gm4*gm5*gm8 + Vinp*g03*g08*gm1*gm2*gm4*gm5*gm7 +  
Vinp*g04*g05*gm1*gm2*gm3*gm7*gm8 + Vinp*g04*g07*gm1*gm2*gm3*gm5*gm8 +  
Vinp*g04*g08*gm1*gm2*gm3*gm5*gm7 + Vinp*g05*g07*gm1*gm2*gm3*gm4*gm8 +  
Vinp*g05*g08*gm1*gm2*gm3*gm4*gm7 + Vinp*g07*g08*gm1*gm2*gm3*gm4*gm5 +  
Vinp*g02*g06*gm1*gm3*gm4*gm7*gm8 + Vinp*g03*g06*gm1*gm2*gm4*gm7*gm8 +  
Vinp*g04*g06*gm1*gm2*gm3*gm7*gm8 + Vinp*g06*g07*gm1*gm2*gm3*gm4*gm8 +  
Vinp*g02*g08*gm1*gm3*gm4*gm7*gm8 + Vinp*g03*g08*gm1*gm2*gm4*gm7*gm8 +  
Vinp*g04*g08*gm1*gm2*gm3*gm7*gm8 + Vinp*g07*g08*gm1*gm2*gm3*gm4*gm8 +  
Vinp*g06*g011*gm1*gm3*gm4*gm5*gm8 + Vinp*g06*g011*gm1*gm3*gm4*gm7*gm8 +  
Vinp*g08*g011*gm1*gm3*gm4*gm7*gm8 + Vinp*g06*gm1*gm2*gm3*gm4*gm5*gm7 +  
Vinp*g06*gm1*gm2*gm3*gm4*gm5*gm8 + Vinp*g02*gm1*gm3*gm4*gm5*gm7*gm8 +  
Vinp*g03*gm1*gm2*gm4*gm5*gm7*gm8 + Vinp*g04*gm1*gm2*gm3*gm5*gm7*gm8 +  
Vinp*g05*gm1*gm2*gm3*gm4*gm7*gm8 + Vinp*g07*gm1*gm2*gm3*gm4*gm5*gm8 +  
Vinp*g08*gm1*gm2*gm3*gm4*gm5*gm7 + Vinp*g06*gm1*gm2*gm3*gm4*gm7*gm8 +  
Vinp*g08*gm1*gm2*gm3*gm4*gm7*gm8 + Vinp*gm1*gm2*gm3*gm4*gm5*gm7*gm8)
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(g01*g02*g03*g04*g07*gm8^2 + g01*g02*g03*g04*gm7*gm8^2 + g01*g02*g04*g07*gm3*gm8^2 +  
g02*g03*g04*g07*gm1*gm3*gm8^2 + g01*g02*g04*gm3*gm7*gm8^2 + g02*g03*g04*gm1*gm7*gm8^2 +  
g02*g04*g07*gm1*gm3*gm8^2 + g02*g04*gm1*gm3*gm7*gm8^2 + g01*g02*g03*g04*g05*g06*g07 +  
g01*g02*g03*g04*g05*g06*g08 + g01*g02*g03*g04*g05*g07*g08 + g01*g02*g03*g04*g06*g07*g08 +  
g01*g02*g03*g04*g05*g08*g011 + g01*g02*g03*g04*g06*g07*g08 + g01*g02*g03*g04*g05*g06*g07*g08 +  
g01*g02*g03*g04*g05*g06*g07*g08 + g01*g02*g03*g04*g07*g08*g011 + g01*g02*g03*g05*g06*g07*g08*g011 +  
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g02*g05*g06*g07*gm8*gm011*gm3 + g03*g04*g05*g06*gm8*gm011*gm7 + g03*g04*g06*g07*gm8*gm011*gm5 +  
g03*g05*g06*g07*gm8*gm011*gm4 + g04*g05*g06*g07*gm8*gm011*gm3 + g01*g02*g03*g04*g06*gm5*gm7 +  
g01*g02*g04*g05*g06*gm3*gm7 + g01*g02*g04*g06*gm3*gm5 + g02*g03*g04*g05*g06*gm1*gm7 +  
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g01*g03*g05*g06*gm8*gm2*gm4 + g02*g03*g04*g05*g06*gm1*gm8 + g01*g02*g03*g04*g05*gm7*gm8 +  
g01*g02*g03*g04*g07*gm5*gm8 + g01*g02*g03*g04*g08*gm5*gm7 + g01*g02*g04*g05*g07*gm3*gm8 +  
g01*g02*g04*g05*g08*gm3*gm7 + g01*g02*g04*g07*gm8*gm3*gm5 + g02*g03*g04*g05*g07*gm1*gm8 +  
g02*g03*g04*g05*g08*gm1*gm7 + g02*g03*g04*g07*gm8*gm1*gm5 + g02*g04*g05*g07*gm8*gm1*gm3 +  
g01*g02*g03*g04*g06*gm7*gm8 + g01*g02*g03*g06*gm8*gm4*gm7 + g01*g02*g04*g06*g07*gm3*gm8 +
```

MATLAB Solution Continued 7:

g01*g03*g04*g06*g08*gm2*gm7 + g01*g03*g06*g07*g08*gm2*gm4 + g02*g03*g04*g06*g07*gm1*gm8 +
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g03*g05*g06*g07*g08*gm1*gm4 + g04*g05*g06*g07*g08*gm1*gm3 + g02*g03*g04*g06*g08*gm5*gm7 +
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g01*g02*g04*g07*g011*gm5*gm8 + g01*g02*g04*g08*g011*gm5*gm7 + g01*g03*g06*g08*g011*gm4*gm5 +
g02*g03*g04*g06*g011*gm5*gm7 + g02*g04*g05*g06*g011*gm3*gm7 + g02*g04*g06*g07*g011*gm3*gm5 +
g01*g02*g06*g08*g011*gm5*gm7 + g01*g03*g06*g08*g011*gm4*gm7 + g02*g03*g04*g05*g011*gm7*gm8 +
g02*g03*g04*g07*g011*gm5*gm8 + g02*g03*g04*g08*g011*gm5*gm7 + g02*g04*g05*g07*g011*gm3*gm8 +
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CL*g01*g02*g03*g04*g05*g06*g07*s + CL*g01*g02*g03*g05*g06*g011*s + CL*g01*g03*g04*g05*g07*g08*s +
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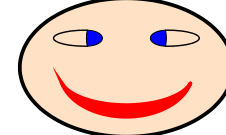
MATLAB Solution Continued 8:

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CL*g01*g04*g05*g06*g07*gm3*s + CL*g02*g03*g05*g07*g08*gm1*s + CL*g03*g04*g05*g06*g07*gm1*s +
CL*g01*g02*g04*g05*g07*gm8*s + CL*g01*g02*g04*g05*g08*gm7*s + CL*g01*g02*g04*g05*g08*gm5*s +
CL*g01*g02*g05*g07*g08*gm4*s + CL*g01*g04*g05*g07*g08*gm2*s + CL*g02*g03*g04*g05*g06*gm7*s +
CL*g02*g03*g04*g06*g07*gm5*s + CL*g02*g03*g05*g06*g07*gm4*s + CL*g02*g04*g05*g06*g07*gm3*s +
CL*g03*g04*g05*g06*g07*gm2*s + CL*g01*g02*g03*g06*g011*gm5*s + CL*g01*g03*g04*g05*g07*gm8*s +
CL*g01*g03*g04*g05*g08*gm7*s + CL*g01*g03*g04*g07*g08*gm5*s + CL*g01*g03*g05*g07*g08*gm4*s +
CL*g01*g04*g05*g07*g08*gm3*s + CL*g03*g04*g05*g07*g08*gm1*s + CL*g02*g03*g04*g05*g07*gm8*s +
CL*g02*g03*g04*g05*g08*gm7*s + CL*g02*g03*g04*g07*g08*gm5*s + CL*g02*g03*g05*g07*g08*gm4*s +
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CL*g01*g02*g03*g06*g011*gm7*s + CL*g01*g02*g03*g08*gm011*gm5*s + CL*g01*g03*g04*g06*g011*gm5*s +
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CL*g01*g04*g07*g08*gm011*gm5*s + CL*g01*g05*g07*g08*gm011*gm4*s + CL*g01*g05*g08*gm011*gm3*s +
CL*g02*g03*g05*g08*gm011*gm7*s + CL*g02*g03*g06*g07*gm011*gm5*s + CL*g02*g03*g06*g07*gm011*gm3*s +
CL*g02*g03*g05*g08*gm011*gm7*s + CL*g02*g03*g06*g07*gm011*gm5*s + CL*g02*g03*g06*g07*gm011*gm3*s +
CL*g03*g04*g05*g06*g07*gm011*gm3*s + CL*g03*g04*g05*g07*gm011*gm8*s + CL*g03*g04*g05*g08*gm011*gm7*s +
CL*g03*g04*g07*g08*gm011*gm5*s + CL*g03*g05*g07*gm08*gm011*gm4*s + CL*g04*g05*g07*gm08*gm011*gm3*s +
CL*g01*g02*g03*g06*gm4*gm5*s + CL*g01*g03*g04*g06*gm2*gm5*s + CL*g01*g03*g05*g06*gm2*gm4*s +
CL*g01*g02*g03*g04*gm5*gm8*s + CL*g01*g02*g03*g05*gm4*gm8*s + CL*g01*g02*g03*g06*gm4*gm7*s +
CL*g01*g02*g03*g08*gm4*gm5*s + CL*g01*g03*g04*g05*gm2*gm8*s + CL*g01*g03*g06*gm7*gm2*gm4*s +
CL*g01*g02*g03*g06*gm5*gm7*s + CL*g01*g03*g05*g08*gm2*gm4*s + CL*g01*g02*g03*g06*gm3*gm7*s +
CL*g01*g02*g03*g05*gm1*gm7*s + CL*g02*g03*g06*g07*gm1*gm5*s + CL*g02*g05*g06*g07*gm1*gm3*s +
CL*g01*g02*g04*g06*gm5*gm7*s + CL*g01*g02*g05*g06*gm4*gm7*s + CL*g01*g02*g06*g07*gm4*gm5*s +
CL*g01*g03*g04*g07*gm2*gm8*s + CL*g01*g03*g04*g08*gm2*gm7*s + CL*g01*g03*g07*gm2*gm4*s +
CL*g01*g04*g05*g06*gm2*gm7*s + CL*g01*g04*g06*g07*gm2*gm5*s + CL*g01*g04*g07*gm2*gm3*s +
CL*g01*g02*g03*g05*gm7*gm8*s + CL*g01*g02*g03*g07*gm5*gm8*s + CL*g01*g02*g03*g08*gm5*gm7*s +
CL*g01*g02*g05*g07*gm3*gm8*s + CL*g01*g02*g05*g08*gm3*gm7*s + CL*g01*g02*g05*g08*gm3*gm5*s +
CL*g01*g03*g04*g06*gm5*gm7*s + CL*g01*g03*g05*g06*gm4*gm7*s + CL*g01*g03*g06*g07*gm4*gm5*s +
CL*g01*g04*g05*g06*gm3*gm7*s + CL*g01*g04*g06*g07*gm3*gm5*s + CL*g01*g04*g07*gm3*gm4*s +
CL*g02*g03*g05*g07*gm1*gm8*s + CL*g02*g03*g05*g08*gm1*gm7*s + CL*g02*g03*g07*gm8*gm1*gm5*s +
CL*g02*g05*g07*gm8*gm1*gm3*s + CL*g03*g04*g05*g06*gm1*gm7*s + CL*g03*g04*g06*g07*gm1*gm5*s +
CL*g01*g02*g04*g07*gm1*gm4*s + CL*g01*g02*g05*g06*gm1*gm3*s + CL*g01*g02*g06*g07*gm1*gm8*s +
CL*g01*g02*g04*g07*gm5*gm8*s + CL*g01*g02*g04*g08*gm5*gm7*s + CL*g01*g02*g05*g07*gm4*gm8*s +
CL*g01*g02*g05*g08*gm4*gm7*s + CL*g01*g02*g07*gm8*gm4*gm5*s + CL*g01*g04*g05*g07*gm2*gm8*s +
CL*g01*g04*g05*g08*gm2*gm7*s + CL*g01*g04*g07*gm8*gm2*gm5*s + CL*g01*g05*g07*gm8*gm2*gm4*s +
CL*g02*g03*g04*g06*gm5*gm7*s + CL*g02*g03*g05*gm4*gm7*s + CL*g02*g03*g06*g07*gm4*gm5*s +
CL*g02*g04*g05*g06*gm3*gm7*s + CL*g02*g04*g06*g07*gm3*gm5*s + CL*g02*g04*g07*gm3*gm4*s +
CL*g03*g04*g05*g06*gm2*gm7*s + CL*g03*g04*g06*gm2*gm5*s + CL*g03*g04*g07*gm2*gm4*s +
CL*g04*g05*g07*gm8*gm2*gm3*s + CL*g01*g02*g03*g011*gm5*gm8*s + CL*g01*g03*g06*g011*gm4*gm5*s +
CL*g01*g02*g03*g05*g011*gm4*gm8*s + CL*g01*g03*g06*g011*gm4*gm7*s + CL*g01*g03*g08*gm011*gm5*gm5*s +
CL*g01*g02*g05*g011*gm7*gm8*s + CL*g01*g02*g07*gm011*gm5*gm8*s + CL*g01*g02*g08*gm011*gm5*gm7*s +
CL*g01*g03*g04*g011*gm7*gm8*s + CL*g01*g03*g07*gm011*gm4*gm8*s + CL*g01*g03*g08*gm011*gm4*gm7*s +
CL*g01*g04*g06*g011*gm5*gm7*s + CL*g01*g05*g06*g011*gm4*gm7*s + CL*g01*g06*g07*gm011*gm4*gm5*s +
CL*g02*g03*g06*g011*gm5*gm7*s + CL*g02*g05*g06*g011*gm3*gm7*s + CL*g02*g05*g07*gm011*gm4*gm5*s +
CL*g01*g04*g05*g011*gm7*gm8*s + CL*g01*g04*g07*gm011*gm5*gm8*s + CL*g01*g04*g08*gm011*gm5*gm7*s +
CL*g01*g05*g07*gm011*gm4*gm8*s + CL*g01*g05*g08*gm011*gm4*gm7*s + CL*g01*g07*gm08*gm011*gm5*gm7*s +
CL*g02*g03*g05*g011*gm7*gm8*s + CL*g02*g03*g07*gm011*gm5*gm8*s + CL*g02*g03*g08*gm011*gm5*gm7*s +
CL*g02*g05*g07*gm011*gm3*gm8*s + CL*g02*g05*g08*gm011*gm3*gm7*s + CL*g02*g07*gm08*gm011*gm3*gm5*s +
CL*g03*g04*g06*g011*gm5*gm7*s + CL*g03*g05*g06*g011*gm4*gm7*s + CL*g03*g06*g07*gm011*gm4*gm5*s +
CL*g04*g05*g06*g011*gm3*gm7*s + CL*g04*g06*g07*gm011*gm3*gm5*s + CL*g05*g06*g07*gm011*gm3*gm4*s +
CL*g03*g04*g05*g011*gm7*gm8*s + CL*g03*g04*g07*gm011*gm5*gm8*s + CL*g03*g04*g08*gm011*gm5*gm7*s +
```

MATLAB Solution Continued 9:

```
CL*g03*g05*g07*g011*gm4*gm8*s + CL*g03*g05*g08*g011*gm4*gm7*s + CL*g03*g07*g08*g011*gm4*gm5*s +
CL*g04*g05*g07*g011*gm3*gm8*s + CL*g04*g05*g08*g011*gm3*gm7*s + CL*g04*g07*g08*g011*gm3*gm5*s +
CL*g05*g07*g08*g011*gm3*gm4*s + CL*g01*g03*g06*gm2*gm4*gm5*s + CL*g01*g02*g03*g04*gm5*gm8*s +
CL*g01*g03*g04*gm2*gm5*gm8*s + CL*g01*g03*g05*gm2*gm4*gm8*s + CL*g01*g03*g06*gm2*gm4*gm7*s +
CL*g01*g03*g08*gm2*gm4*gm5*s + CL*g01*g02*g06*gm3*gm5*gm7*s + CL*g02*g03*g06*gm2*gm4*gm8*s +
CL*g02*g05*g06*gm1*gm3*gm7*s + CL*g02*g06*g07*gm1*gm3*gm5*s + CL*g01*g02*g03*gm4*gm7*gm8*s +
CL*g01*g02*g06*gm4*gm5*gm7*s + CL*g01*g03*g04*gm2*gm7*gm8*s + CL*g01*g03*g07*gm2*gm4*gm8*s +
CL*g01*g03*g08*gm2*gm4*gm7*s + CL*g01*g04*g06*gm2*gm5*gm7*s + CL*g01*g05*g06*gm2*gm4*gm7*s +
CL*g01*g06*g07*gm2*gm4*gm5*s + CL*g01*g02*g03*gm5*gm7*gm8*s + CL*g01*g02*g05*gm2*gm4*gm7*gm8*s +
CL*g01*g02*g07*gm3*gm5*gm8*s + CL*g01*g02*g08*gm3*gm5*gm7*s + CL*g01*g03*g06*gm4*gm5*gm7*s +
CL*g01*g04*g06*gm3*gm5*gm7*s + CL*g01*g05*g06*gm3*gm4*gm7*s + CL*g01*g06*g07*gm3*gm4*gm5*s +
CL*g02*g03*g05*gm1*gm7*gm8*s + CL*g02*g03*g07*gm1*gm5*gm8*s + CL*g02*g03*g08*gm1*gm5*gm7*s +
CL*g02*g05*g07*gm1*gm3*gm8*s + CL*g02*g05*g08*gm1*gm3*gm7*s + CL*g02*g07*g08*gm1*gm3*gm5*s +
CL*g03*g04*g06*gm1*gm5*gm7*s + CL*g03*g05*g06*gm1*gm4*gm7*s + CL*g03*g06*g07*gm1*gm4*gm5*s +
CL*g04*g05*g06*gm1*gm3*gm7*s + CL*g04*g06*g07*gm1*gm3*gm5*s + CL*g05*g06*g07*gm1*gm3*gm4*s +
CL*g01*g02*g04*gm5*gm7*gm8*s + CL*g01*g02*g05*gm4*gm7*gm8*s + CL*g01*g02*g07*gm4*gm5*gm8*s +
CL*g01*g04*g08*gm2*gm5*gm7*s + CL*g01*g05*g07*gm2*gm4*gm8*s + CL*g01*g05*g08*gm2*gm4*gm7*s +
CL*g01*g07*g08*gm2*gm4*gm5*s + CL*g02*g03*g06*gm4*gm5*gm7*s + CL*g02*g04*g06*gm3*gm5*gm7*s +
CL*g02*g05*g06*gm3*gm4*gm7*s + CL*g02*g06*g07*gm3*gm4*gm5*s + CL*g03*g04*g06*gm2*gm5*gm7*s +
CL*g03*g05*g06*gm2*gm4*gm7*s + CL*g03*g06*g07*gm2*gm3*gm8*s + CL*g04*g05*g06*gm2*gm3*gm7*s +
CL*g04*g06*g07*gm2*gm3*gm5*s + CL*g05*g06*g07*gm2*gm3*gm4*s + CL*g01*g03*g04*gm5*gm7*gm8*s +
CL*g01*g03*g05*gm4*gm7*gm8*s + CL*g01*g03*g08*gm4*gm5*gm7*s + CL*g01*g04*g08*gm3*gm5*gm7*s +
CL*g01*g05*g07*gm3*gm4*gm8*s + CL*g01*g05*g08*gm3*gm4*gm5*s + CL*g01*g07*g08*gm3*gm4*gm5*s +
CL*g03*g04*g05*gm1*gm7*gm8*s + CL*g03*g04*g07*gm1*gm5*gm8*s + CL*g03*g04*g08*gm1*gm5*gm7*s +
CL*g03*g05*g07*gm1*gm4*gm8*s + CL*g03*g05*g08*gm1*gm4*gm5*s + CL*g03*g07*g08*gm1*gm4*gm5*s +
CL*g04*g05*g08*gm2*gm3*gm7*s + CL*g04*g07*g08*gm2*gm3*gm5*s + CL*g05*g07*g08*gm2*gm3*gm4*s +
CL*g01*g03*g011*gm4*gm5*gm8*s + CL*g01*g06*g011*gm4*gm5*gm7*s + CL*g02*g02*g011*gm4*gm5*gm8*s +
CL*g01*g06*g011*gm4*gm5*gm7*s + CL*g02*g02*g06*g011*gm3*gm5*gm7*s + CL*g01*g04*g011*gm5*gm7*gm8*s +
CL*g01*g08*g011*gm4*gm5*gm8*s + CL*g01*g08*g011*gm4*gm5*gm7*s + CL*g01*g08*g011*gm4*gm5*gm6*s +
CL*g02*g03*g011*gm5*gm7*gm8*s + CL*g02*g05*g011*gm3*gm7*gm8*s + CL*g02*g07*g011*gm3*gm5*gm8*s +
CL*g02*g08*g011*gm3*gm5*gm7*s + CL*g03*g06*g011*gm4*gm5*gm7*s + CL*g04*g06*g011*gm4*gm5*gm7*s +
CL*g03*g05*g011*gm4*gm7*gm8*s + CL*g03*g07*g011*gm3*gm4*gm5*s + CL*g03*g04*g011*gm5*gm7*gm8*s +
CL*g03*g08*g011*gm4*gm5*gm8*s + CL*g03*g08*g011*gm4*gm5*gm7*s + CL*g03*g08*g011*gm4*gm5*gm6*s +
CL*g04*g05*g011*gm3*gm7*gm8*s + CL*g04*g07*g011*gm3*gm5*gm8*s + CL*g04*g08*g011*gm3*gm5*gm7*s +
CL*g05*g07*g011*gm3*gm4*gm8*s + CL*g05*g08*g011*gm3*gm4*gm7*s + CL*g07*g08*g011*gm3*gm4*gm5*s +
CL*g01*g03*gm2*gm4*gm5*gm8*s + CL*g02*g06*gm1*gm3*gm5*gm7*s + CL*g01*g03*gm2*gm4*gm7*gm8*s +
CL*g01*g06*gm2*gm4*gm5*gm7*s + CL*g01*g02*gm3*gm5*gm7*gm8*s + CL*g01*g06*gm3*gm4*gm5*gm7*s +
CL*g02*g03*gm1*gm5*gm7*gm8*s + CL*g02*g05*gm1*gm3*gm7*gm8*s + CL*g02*g07*gm1*gm3*gm5*gm8*s +
CL*g02*g08*gm1*gm3*gm5*gm7*s + CL*g03*g06*gm1*gm4*gm5*gm7*s + CL*g04*g06*gm1*gm4*gm5*gm7*s +
CL*g05*gm2*gm3*gm4*gm7*gm8*s + CL*g07*gm2*gm3*gm4*gm5*gm8*s + CL*g08*gm2*gm3*gm4*gm5*gm7*s +
CL*g011*gm3*gm4*gm5*gm7*gm8*s + CL*gm1*gm3*gm4*gm5*gm7*gm8*s + CL*gm2*gm3*gm4*gm5*gm7*gm8*s)
```

We now (finally) have a complete analytic expression !



Individual product terms have product of 7 small-signal parameters

- Approximately 2000 factors in output characteristics
- Approximately 14,000 small-signal parameter appearances
- GB expression much longer

$$\frac{V_{in}p \cdot g_{08} \cdot g_{m1} \cdot g_{m2} \cdot g_{m3} \cdot g_{m4} \cdot g_{m5} \cdot g_{m7} + V_{in}p \cdot g_{06} \cdot g_{m1} \cdot g_{m2} \cdot g_{m3} \cdot g_{m4} \cdot g_{m7} \cdot g_{m8} + V_{in}p \cdot g_{08} \cdot g_{m1} \cdot g_{m2} \cdot g_{m3} \cdot g_{m4} \cdot g_{m7} \cdot g_{m8} + V_{in}p \cdot g_{m1} \cdot g_{m2} \cdot g_{m3} \cdot g_{m4} \cdot g_{m5} \cdot g_{m7} \cdot g_{m8}}{(g_{01} \cdot g_{02} \cdot g_{03} \cdot g_{04} \cdot g_{07} \cdot g_{m8}^2 + g_{01} \cdot g_{02} \cdot g_{03} \cdot g_{04} \cdot g_{m7} \cdot g_{m8}^2 + g_{01} \cdot g_{02} \cdot g_{04} \cdot g_{07} \cdot g_{m3} \cdot g_{m8}^2 + g_{02} \cdot g_{03} \cdot g_{04} \cdot g_{07} \cdot g_{m1} \cdot g_{m8}^2 + g_{01} \cdot g_{02} \cdot g_{04} \cdot g_{m3} \cdot g_{m7} \cdot g_{m8}^2 + g_{02} \cdot g_{03} \cdot g_{04} \cdot g_{m1} \cdot g_{m7} \cdot g_{m8}^2 +$$

$$CL \cdot g_{02} \cdot g_{m3} \cdot g_{m4} \cdot g_{m5} \cdot g_{m7} \cdot g_{m8} \cdot s + CL \cdot g_{03} \cdot g_{m2} \cdot g_{m4} \cdot g_{m5} \cdot g_{m7} \cdot g_{m8} \cdot s + CL \cdot g_{04} \cdot g_{m2} \cdot g_{m3} \cdot g_{m5} \cdot g_{m7} \cdot g_{m8} \cdot s + CL \cdot g_{05} \cdot g_{m2} \cdot g_{m3} \cdot g_{m4} \cdot g_{m7} \cdot g_{m8} \cdot s + CL \cdot g_{07} \cdot g_{m2} \cdot g_{m3} \cdot g_{m4} \cdot g_{m5} \cdot g_{m8} \cdot s + CL \cdot g_{08} \cdot g_{m2} \cdot g_{m3} \cdot g_{m4} \cdot g_{m5} \cdot g_{m7} \cdot s + CL \cdot g_{011} \cdot g_{m3} \cdot g_{m4} \cdot g_{m5} \cdot g_{m7} \cdot g_{m8} \cdot s + CL \cdot g_{m1} \cdot g_{m3} \cdot g_{m4} \cdot g_{m5} \cdot g_{m7} \cdot g_{m8} \cdot s + CL \cdot g_{m2} \cdot g_{m3} \cdot g_{m4} \cdot g_{m5} \cdot g_{m7} \cdot g_{m8} \cdot s)$$

MATLAB simulation results using Symbolic Math Toolbox

Differential Analysis with $V_d = V_{in}^+ - V_{in}^-$

MATLAB Solution:

```
GainAv =
(g01*g03*g04*g07*gm2*gm8^2 + g02*g03*g04*g07*gm1*gm8^2 + g02*g03*g07*g011*gm1*gm8^2 +
g03*g04*g07*g011*gm1*gm8^2 + g01*g03*g04*gm2*gm7*gm8^2 + g01*g03*g07*gm2*gm4*gm8^2 +
g01*g04*g07*gm2*gm3*gm8^2 + g02*g03*g04*gm1*gm7*gm8^2 + g02*g03*g07*gm1*gm4*gm8^2 +
g02*g04*g07*gm1*gm3*gm8^2 + 2*g03*g04*g07*gm1*gm2*gm8^2 + g02*g03*g011*gm1*gm7*gm8^2 +
g02*g07*g011*gm1*gm3*gm8^2 + g03*g04*g011*gm1*gm7*gm8^2 + g03*g07*g011*gm1*gm4*gm8^2 +
g04*g07*g011*gm1*gm3*gm8^2 + g01*g03*gm2*gm4*gm7*gm8^2 + g01*g04*gm2*gm3*gm7*gm8^2 +
g01*g07*gm2*gm3*gm4*gm8^2 + g02*g03*gm1*gm4*gm7*gm8^2 + g02*g04*gm1*gm3*gm7*gm8^2 +
g02*g07*gm1*gm3*gm4*gm8^2 + 2*g03*g04*gm1*gm2*gm7*gm8^2 + 2*g03*g07*gm1*gm2*gm4*gm8^2 +
2*g04*g07*gm1*gm2*gm3*gm8^2 + g02*g011*gm1*gm3*gm7*gm8^2 + g03*g011*gm1*gm4*gm7*gm8^2 +
g04*g011*gm1*gm3*gm7*gm8^2 + g07*g011*gm1*gm3*gm4*gm8^2 + g01*gm2*gm3*gm4*gm7*gm8^2 +
g02*gm1*gm3*gm4*gm7*gm8^2 + 2*g03*gm1*gm2*gm4*gm7*gm8^2 + 2*g04*gm1*gm2*gm3*gm7*gm8^2 +
2*g07*gm1*gm2*gm3*gm4*gm8^2 + g011*gm1*gm3*gm4*gm7*gm8^2 + 2*gm1*gm2*gm3*gm4*gm7*gm8^2 +
g01*g03*g04*g05*g06*g07*gm2 + g02*g03*g04*g05*g06*g07*gm1 + g01*g03*g04*g05*g07*g08*gm2 +
g02*g03*g04*g05*g07*g08*gm1 + g01*g03*g04*g05*g06*g011*gm2 + g01*g03*g04*g05*g08*g011*gm2 +
g01*g03*g04*g06*g07*g011*gm2 + g01*g03*g04*g07*g08*g011*gm2 + g01*g04*g05*g06*g07*g011*gm2 +
g01*g04*g05*g07*g08*g011*gm2 + g03*g04*g05*g06*g07*g011*gm2 + g03*g04*g05*g07*g08*g011*gm2 +
g01*g03*g04*g05*g06*gm2*gm7 + g01*g03*g04*g06*g07*gm2*gm5 + g01*g03*g05*g06*g07*gm2*gm4 +
g01*g04*g05*g06*g07*gm2*gm3 + g02*g03*g04*g05*g06*gm1*gm7 + g02*g03*g04*g06*g07*gm1*gm5 +
g02*g03*g05*g06*g07*gm1*gm4 + g02*g04*g05*g06*g07*gm1*gm3 + 2*g03*g04*g05*g06*g07*gm1*gm2 +
g01*g03*g04*g05*g08*gm2*gm7 + g02*g03*g04*g05*g06*gm1*gm8 + g01*g03*g04*g05*g07*gm2*gm8 +
g01*g03*g04*g05*g08*gm2*gm7 + g01*g03*g04*g07*g08*gm2*gm5 + g01*g03*g05*g07*g08*gm2*gm4 +
g01*g04*g05*g07*g08*gm2*gm3 + g02*g03*g04*g05*g07*gm1*gm8 + g02*g03*g04*g05*g08*gm1*gm7 +
g02*g03*g04*g07*g08*gm1*gm5 + g02*g03*g05*g07*g08*gm1*gm4 + g02*g04*g05*g07*g08*gm1*gm3 +
2*g03*g04*g05*g07*g08*gm1*gm2 + g01*g03*g04*g06*g07*gm2*gm8 + g02*g03*g04*g06*g07*gm1*gm8 +
g01*g03*g04*g06*g07*g08*gm2*gm8 + g01*g03*g05*g06*g011*gm2*gm4 + g01*g03*g04*g07*g08*gm2*gm8 +
g02*g03*g04*g07*g08*gm1*gm8 + g01*g03*g04*g05*g011*gm2*gm5 + g01*g03*g04*g06*g011*gm2*gm7 +
g01*g03*g04*g08*g011*gm2*gm5 + g01*g03*g05*g08*g011*gm2*gm4 + g01*g03*g06*g07*g011*gm2*gm4 +
g01*g03*g04*g07*g011*gm2*gm8 + g01*g03*g04*g08*g011*gm2*gm7 + g01*g03*g07*g08*g011*gm2*gm4 +
g01*g04*g05*g06*g011*gm2*gm7 + g01*g04*g06*g07*g011*gm2*gm5 + g01*g04*g05*g08*g011*gm2*gm7 +
g01*g04*g07*g08*g011*gm2*gm5 + g01*g05*g07*g08*g011*gm2*gm4 + g02*g03*g06*g07*g011*gm1*gm8 +
g03*g04*g05*g06*g011*gm1*gm8 + g03*g04*g05*g06*g011*gm2*gm7 + g03*g04*g06*g07*g011*gm2*gm5 +
g03*g05*g06*g07*g011*gm2*gm4 + g04*g05*g06*g07*g011*gm2*gm3 + g02*g03*g07*g08*g011*gm1*gm8 +
g03*g04*g05*g07*g011*gm2*gm8 + g03*g04*g05*g08*g011*gm2*gm7 + g03*g04*g06*g07*g011*gm1*gm8 +
g03*g04*g07*g08*g011*gm2*gm5 + g03*g05*g07*g08*g011*gm2*gm4 + g04*g05*g07*g08*g011*gm2*gm3 +
g03*g04*g07*g08*g011*gm1*gm8 + g01*g03*g04*g06*gm2*gm5*gm7 + g01*g03*g05*g06*gm2*gm4*gm7 +
g01*g03*g06*g07*gm2*gm3*gm5 + g01*g04*g05*g06*gm2*gm3*gm7 + g01*g04*g06*g07*gm2*gm3*gm5 +
g01*g05*g06*g07*gm2*gm3*gm4 + g02*g03*g04*g06*gm1*gm5*gm7 + g02*g03*g05*g06*gm1*gm4*gm7 +
g02*g03*g06*g07*gm1*gm4*gm5 + g02*g04*g05*g06*gm1*gm3*gm7 + g02*g04*g06*g07*gm1*gm3*gm5 +
g02*g05*g06*g07*gm1*gm3*gm4 + 2*g03*g04*g05*g06*gm1*gm2*gm7 + 2*g03*g04*g06*g07*gm1*gm2*gm5 +
2*g03*g05*g06*g07*gm1*gm2*gm4 + 2*g04*g05*g06*g07*gm1*gm2*gm3 + g01*g03*g04*g06*gm2*gm5*gm8 +
g01*g03*g05*g06*gm2*gm4*gm8 + g01*g04*g05*g06*gm2*gm3*gm8 + g02*g03*g04*g06*gm1*gm5*gm8 +
g02*g03*g05*g06*gm1*gm4*gm8 + g02*g04*g05*g06*gm1*gm3*gm8 + 2*g03*g04*g05*g06*gm1*gm2*gm8 +
g01*g03*g04*g05*gm2*gm7*gm8 + g01*g03*g04*g08*gm2*gm5*gm7 + g01*g03*g04*g08*gm2*gm4*gm5 +
g01*g03*g05*gm2*gm4*gm8 + g01*g03*g07*g08*gm2*gm4*gm5 + g01*g03*g07*g08*gm2*gm3*gm5 +
g01*g04*g05*g07*gm2*gm3*gm8 + g01*g04*g05*g08*gm2*gm3*gm7 + g01*g04*g07*g08*gm2*gm3*gm5 +
g01*g05*g07*g08*gm2*gm3*gm4 + g02*g03*g04*g05*gm1*gm7*gm8 + g02*g03*g04*g07*gm1*gm5*gm8 +
g02*g03*g04*g08*gm1*gm5*gm7 + g02*g03*g05*g07*gm1*gm4*gm8 + g02*g03*g05*g08*gm1*gm4*gm7 +
g02*g03*g07*g08*gm1*gm4*gm5 + g02*g04*g05*g07*gm1*gm3*gm8 + g02*g04*g05*g08*gm1*gm3*gm7 +
g02*g04*g07*g08*gm1*gm3*gm5 + g02*g05*g07*g08*gm1*gm3*gm4 + 2*g03*g04*g05*g07*gm1*gm2*gm8 +
2*g03*g04*g05*g08*gm1*gm2*gm7 + 2*g03*g04*g06*g07*gm1*gm2*gm5 + 2*g03*g05*g07*g08*gm1*gm2*gm4 +
2*g04*g05*g07*g08*gm1*gm2*gm3 + g01*g03*g04*g06*gm2*gm7*gm8 + g01*g03*g06*g07*gm2*gm4*gm8 +
g01*g04*g06*g07*gm2*gm3*gm8 + g02*g03*g04*g06*gm1*gm7*gm8 + g02*g03*g06*g07*gm1*gm4*gm8 +
g02*g04*g06*g07*gm1*gm3*gm8 + 2*g03*g04*g06*g07*gm1*gm2*gm8 + g01*g03*g06*g011*gm2*gm4*gm5 +
g01*g03*g04*g08*gm2*gm7*gm8 + g01*g03*g07*g08*gm2*gm4*gm8 + g01*g04*g07*g08*gm2*gm3*gm8 +
g02*g03*g04*g08*gm1*gm7*gm8 + g02*g03*g07*g08*gm1*gm4*gm8 + g02*g04*g07*g08*gm1*gm3*gm8 +
2*g03*g04*g07*g08*gm1*gm2*gm8 + g01*g03*g04*g011*gm2*gm5*gm8 + g01*g03*g05*g011*gm2*gm4*gm8 +
g01*g03*g06*g011*gm2*gm4*gm7 + g01*g03*g08*g011*gm2*gm4*gm5 + g01*g03*g04*g011*gm2*gm7*gm8 +
g01*g03*g07*g011*gm2*gm4*gm8 + g01*g03*g08*g011*gm2*gm4*gm7 + g01*g04*g06*g011*gm2*gm5*gm7 +
g01*g05*g06*g011*gm2*gm4*gm7 + g01*g06*g07*g011*gm2*gm4*gm5 + g02*g03*g06*g011*gm1*gm5*gm8 +
g02*g05*g06*g011*gm1*gm3*gm8 + g01*g04*g05*g011*gm2*gm7*gm8 + g01*g04*g07*g011*gm2*gm5*gm8 +
g01*g04*g08*g011*gm2*gm5*gm7 + g01*g05*g07*g011*gm2*gm4*gm8 + g01*g05*g08*g011*gm2*gm4*gm7 +
g01*g07*g08*g011*gm2*gm4*gm5 + g02*g03*g06*g011*gm1*gm7*gm8 + g02*g06*g07*g011*gm1*gm3*gm8 +
g03*g04*g06*g011*gm1*gm5*gm8 + g03*g04*g06*g011*gm2*gm5*gm7 + g03*g05*g06*g011*gm1*gm4*gm8 +
g03*g05*g06*g011*gm2*gm4*gm7 + g03*g06*g07*g011*gm2*gm4*gm5 + g04*g05*g06*g011*gm1*gm3*gm8 +
g04*g05*g06*g011*gm2*gm3*gm7 + g04*g06*g07*g011*gm2*gm3*gm5 + g05*g06*g07*g011*gm2*gm3*gm4 +
g02*g03*g08*g011*gm1*gm7*gm8 + g02*g07*g08*g011*gm1*gm3*gm8 + g03*g04*g05*g011*gm2*gm7*gm8 +
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MATLAB Solution Continued 1:

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g03*g04*g06*g011*gm1*gm7*gm8 + g03*g04*g07*g011*gm2*gm5*gm8 + g03*g04*g08*g011*gm2*gm5*gm7 +
g03*g05*g07*g011*gm2*gm4*gm8 + g03*g05*g08*g011*gm2*gm4*gm7 + g03*g06*g07*g011*gm1*gm4*gm8 +
g03*g07*g08*g011*gm2*gm4*gm5 + g04*g05*g07*g011*gm2*gm3*gm8 + g04*g05*g08*g011*gm2*gm3*gm7 +
g04*g06*g07*g011*gm1*gm3*gm8 + g04*g07*g08*g011*gm2*gm3*gm5 + g05*g07*g08*g011*gm2*gm3*gm4 +
g03*g04*g08*g011*gm1*gm7*gm8 + g03*g07*g08*g011*gm1*gm4*gm8 + g04*g07*g08*g011*gm1*gm3*gm8 +
g01*g03*g06*gm2*gm4*gm5*gm7 + g01*g04*g06*gm2*gm3*gm5*gm7 + g01*g05*g06*gm2*gm3*gm4*gm7 +
g01*g06*g07*gm2*gm3*gm4*gm5 + g02*g03*g06*gm1*gm4*gm5*gm7 + g02*g04*g06*gm1*gm3*gm5*gm7 +
g02*g05*g06*gm1*gm3*gm4*gm7 + g02*g06*g07*gm1*gm3*gm4*gm5 + 2*g03*g04*g06*gm1*gm2*gm5*gm7 +
2*g03*g05*g06*gm1*gm2*gm4*gm7 + 2*g03*g06*g07*gm1*gm2*gm4*gm5 + 2*g04*g05*g06*gm1*gm2*gm3*gm7 +
2*g04*g06*g07*gm1*gm2*gm3*gm5 + 2*g05*g06*g07*gm1*gm2*gm3*gm4 + g01*g03*g06*gm2*gm4*gm5*gm8 +
g01*g04*g06*gm2*gm3*gm5*gm8 + g01*g05*g06*gm2*gm3*gm4*gm8 + g02*g03*g06*gm1*gm4*gm5*gm8 +
g02*g04*g06*gm1*gm3*gm5*gm8 + g02*g05*g06*gm1*gm3*gm4*gm8 + 2*g03*g04*g06*gm1*gm2*gm5*gm8 +
2*g03*g05*g06*gm1*gm2*gm4*gm8 + 2*g04*g05*g06*gm1*gm2*gm3*gm8 + g01*g03*g04*gm2*gm5*gm7*gm8 +
g01*g03*g05*gm2*gm4*gm7*gm8 + g01*g03*g07*gm2*gm4*gm5*gm8 + g01*g03*g08*gm2*gm4*gm5*gm7 +
g01*g04*g05*gm2*gm3*gm5*gm8 + g01*g04*g07*gm2*gm3*gm5*gm8 + g01*g04*g08*gm2*gm3*gm5*gm7 +
g01*g05*g07*gm2*gm3*gm4*gm8 + g01*g05*g08*gm2*gm3*gm4*gm7 + g01*g07*g08*gm2*gm3*gm4*gm5 +
g02*g03*g04*gm1*gm3*gm7*gm8 + g02*g03*g05*gm1*gm4*gm7*gm8 + g02*g03*g07*gm1*gm4*gm5*gm8 +
g02*g03*g08*gm1*gm4*gm5*gm7 + g02*g04*g05*gm1*gm3*gm7*gm8 + g02*g04*g07*gm1*gm3*gm5*gm8 +
g02*g04*g08*gm1*gm3*gm5*gm7 + g02*g05*g07*gm1*gm3*gm4*gm8 + g02*g05*g08*gm1*gm3*gm4*gm7 +
g02*g07*g08*gm1*gm3*gm4*gm5 + 2*g03*g04*g05*gm1*gm2*gm7*gm8 + 2*g03*g04*g07*gm1*gm2*gm5*gm8 +
2*g03*g04*g08*gm1*gm2*gm5*gm7 + 2*g03*g05*g07*gm1*gm2*gm4*gm8 + 2*g03*g05*g08*gm1*gm2*gm3*gm7 +
2*g04*g07*g08*gm1*gm2*gm3*gm5 + 2*g05*g07*g08*gm1*gm2*gm3*gm4 + g01*g03*g06*gm2*gm4*gm7*gm8 +
g01*g04*g06*gm2*gm3*gm7*gm8 + g02*g04*g06*gm1*gm3*gm4*gm8 + g02*g03*g06*gm1*gm2*gm7*gm8 +
2*g03*g06*g07*gm1*gm3*gm4*gm8 + 2*g04*g06*g07*gm1*gm2*gm3*gm8 + g01*g03*g08*gm2*gm4*gm7*gm8 +
g01*g04*g08*gm2*gm3*gm7*gm8 + g01*g07*g08*gm2*gm3*gm4*gm8 + g02*g03*g08*gm1*gm4*gm7*gm8 +
g02*g04*g08*gm1*gm3*gm7*gm8 + 2*g03*g04*g08*gm1*gm2*gm7*gm8 + 2*g03*g04*g08*gm1*gm2*gm7*gm8 +
2*g03*g07*g08*gm1*gm2*gm4*gm8 + g01*g03*g04*gm2*gm5*gm8 + g01*g03*g07*gm1*gm2*gm5*gm8 +
g01*g04*g011*gm2*gm4*gm7*gm8 + g01*g05*g011*gm2*gm4*gm7*gm8 + g01*g07*g011*gm2*gm4*gm5*gm8 +
g01*g08*g011*gm2*gm4*gm5*gm7 + g02*g06*g011*gm1*gm3*gm5*gm8 + g03*g06*g011*gm1*gm4*gm5*gm8 +
g03*g06*g011*gm2*gm3*gm5*gm7 + g04*g06*g011*gm2*gm3*gm5*gm8 + g06*g07*g011*gm2*gm3*gm4*gm5 +
g02*g08*g011*gm1*gm3*gm4*gm8 + g03*g04*g011*gm2*gm5*gm7*gm8 + g03*g05*g011*gm2*gm4*gm7*gm8 +
g03*g06*g011*gm1*gm4*gm7*gm8 + g04*g07*g011*gm2*gm3*gm5*gm8 + g04*g08*g011*gm2*gm3*gm4*gm7 +
g04*g05*g011*gm2*gm3*gm7*gm8 + g04*g06*g011*gm1*gm3*gm7*gm8 + g04*g07*g011*gm2*gm3*gm5*gm8 +
g04*g08*g011*gm2*gm3*gm5*gm7 + g05*g07*g011*gm2*gm3*gm4*gm8 + g05*g08*g011*gm2*gm3*gm4*gm7 +
g06*g07*g011*gm1*gm3*gm4*gm8 + g07*g08*g011*gm2*gm3*gm4*gm5 + g03*g08*g011*gm1*gm4*gm7*gm8 +
g04*g08*g011*gm1*gm3*gm7*gm8 + g07*g08*g011*gm1*gm3*gm4*gm8 + g01*g06*gm2*gm3*gm4*gm5*gm7 +
g02*g06*gm1*gm3*gm4*gm5*gm7 + 2*g03*g06*gm1*gm2*gm4*gm5*gm7 + 2*g04*g06*gm1*gm2*gm3*gm5*gm7 +
2*g05*g06*gm1*gm2*gm3*gm4*gm5 + 2*g06*g07*gm1*gm2*gm3*gm4*gm5 + g01*g06*gm2*gm3*gm4*gm5*gm8 +
g02*g06*gm1*gm3*gm4*gm5*gm8 + 2*g03*g06*gm1*gm2*gm4*gm5*gm8 + 2*g04*g06*gm1*gm2*gm3*gm5*gm8 +
2*g05*g06*gm1*gm2*gm3*gm4*gm8 + g01*g03*gm2*gm4*gm5*gm7*gm8 + g01*g04*gm2*gm3*gm5*gm7*gm8 +
g01*g05*gm2*gm3*gm4*gm7*gm8 + g01*g07*gm2*gm3*gm4*gm5*gm8 + g01*g08*gm2*gm3*gm4*gm5*gm7 +
g02*g03*gm1*gm4*gm5*gm7*gm8 + g02*g04*gm1*gm3*gm5*gm7*gm8 + g02*g05*gm1*gm3*gm4*gm7*gm8 +
g02*g07*gm1*gm3*gm4*gm5*gm8 + g02*g08*gm1*gm3*gm4*gm5*gm7 + 2*g03*g04*gm1*gm2*gm5*gm7*gm8 +
g02*g03*g05*gm1*gm2*gm4*gm7*gm8 + 2*g03*g07*gm1*gm2*gm4*gm5*gm8 + 2*g03*g08*gm1*gm2*gm4*gm5*gm7 +
2*g04*g05*gm1*gm2*gm3*gm7*gm8 + 2*g04*g07*gm1*gm2*gm3*gm5*gm8 + 2*g04*g08*gm1*gm2*gm3*gm5*gm7 +
2*g05*g07*gm1*gm2*gm3*gm4*gm8 + 2*g05*g08*gm1*gm2*gm3*gm4*gm7 + 2*g07*g08*gm1*gm2*gm3*gm4*gm5 +
2*g08*g011*gm2*gm3*gm4*gm5*gm7 + g08*g011*gm1*gm3*gm4*gm7*gm8 + 2*g06*gm1*gm2*gm3*gm4*gm5*gm7 +
2*g06*gm1*gm2*gm3*gm4*gm5*gm8 + g01*gm2*gm3*gm4*gm5*gm7*gm8 + g02*gm1*gm3*gm4*gm5*gm7*gm8 +
2*g03*gm1*gm2*gm4*gm5*gm7*gm8 + 2*g04*gm1*gm2*gm3*gm5*gm7*gm8 + 2*g05*gm1*gm2*gm3*gm4*gm7*gm8 +
2*g07*gm1*gm2*gm3*gm4*gm5*gm8 + 2*g08*gm1*gm2*gm3*gm4*gm5*gm7 + 2*g06*gm1*gm2*gm3*gm4*gm7*gm8 +
2*g08*gm1*gm2*gm3*gm4*gm7*gm8 + g011*gm2*gm3*gm4*gm5*gm7*gm8 + 2*gm1*gm2*gm3*gm4*gm5*gm7*gm8)
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((2*CL*g01*g02*g03*g04*g05*g06 + 2*CL*g01*g02*g03*g04*g05*g08 + 2*CL*g01*g02*g03*g04*g06*g07 +
2*CL*g01*g02*g03*g05*g06*g07 + 2*CL*g01*g02*g03*g04*g07*g08 + 2*CL*g01*g02*g04*g05*g06*g07 +
2*CL*g01*g02*g03*g05*g07*g08 + 2*CL*g01*g03*g04*g05*g06*g07 + 2*CL*g01*g02*g04*g05*g07*g08 +
2*CL*g02*g03*g04*g05*g06*g07 + 2*CL*g01*g02*g03*g05*g06*g011 + 2*CL*g01*g03*g04*g05*g07*g08 +
2*CL*g02*g03*g04*g05*g07*g08 + 2*CL*g01*g02*g03*g05*g08*g011 + 2*CL*g01*g02*g03*g06*g07*g011 +
2*CL*g01*g03*g04*g05*g06*g011 + 2*CL*g01*g02*g03*g07*g08*g011 + 2*CL*g01*g02*g05*g06*g07*g011 +
2*CL*g01*g03*g04*g05*g08*g011 + 2*CL*g01*g03*g04*g06*g07*g011 + 2*CL*g01*g02*g05*g07*g08*g011 +
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MATLAB Solution Continued 3:

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2*CL*g01*g02*g05*g011*gm7*gm8 + 2*CL*g01*g02*g07*g011*gm5*gm8 + 2*CL*g01*g02*g08*g011*gm5*gm7 +
2*CL*g01*g03*g04*g011*gm7*gm8 + 2*CL*g01*g03*g07*g011*gm4*gm8 + 2*CL*g01*g03*g08*g011*gm4*gm7 +
2*CL*g01*g04*g06*g011*gm5*gm7 + 2*CL*g01*g05*g06*g011*gm4*gm7 + 2*CL*g01*g06*g07*g011*gm4*gm5 +
2*CL*g02*g03*g06*g011*gm5*gm7 + 2*CL*g02*g05*g06*g011*gm3*gm7 + 2*CL*g02*g06*g07*g011*gm3*gm5 +
2*CL*g01*g04*g05*g011*gm7*gm8 + 2*CL*g01*g04*g07*g011*gm5*gm8 + 2*CL*g01*g04*g08*g011*gm5*gm7 +
2*CL*g01*g05*g07*g011*gm4*gm8 + 2*CL*g01*g05*g08*g011*gm4*gm7 + 2*CL*g01*g07*g08*g011*gm4*gm5 +
2*CL*g02*g03*g05*g011*gm7*gm8 + 2*CL*g02*g03*g07*g011*gm5*gm8 + 2*CL*g02*g03*g08*g011*gm5*gm7 +
2*CL*g02*g05*g07*g011*gm3*gm8 + 2*CL*g02*g05*g08*g011*gm3*gm7 + 2*CL*g02*g07*g08*g011*gm3*gm5 +
2*CL*g03*g04*g06*g011*gm5*gm7 + 2*CL*g03*g05*g06*g011*gm4*gm7 + 2*CL*g03*g06*g07*g011*gm4*gm5 +
2*CL*g04*g05*g06*g011*gm3*gm7 + 2*CL*g04*g06*g07*g011*gm3*gm5 + 2*CL*g05*g06*g07*g011*gm3*gm4 +
2*CL*g03*g04*g05*g011*gm7*gm8 + 2*CL*g03*g04*g07*g011*gm5*gm8 + 2*CL*g03*g04*g08*g011*gm5*gm7 +
2*CL*g03*g05*g07*g011*gm4*gm8 + 2*CL*g03*g05*g08*g011*gm4*gm7 + 2*CL*g03*g07*g08*g011*gm4*gm5 +
2*CL*g04*g05*g07*g011*gm3*gm8 + 2*CL*g04*g05*g08*g011*gm3*gm7 + 2*CL*g04*g07*g08*g011*gm3*gm5 +
2*CL*g05*g07*g08*g011*gm3*gm4 + 2*CL*g01*g03*g06*gm2*gm4*gm5 + 2*CL*g01*g02*g03*gm4*gm5*gm8 +
2*CL*g01*g03*g04*gm2*gm5*gm8 + 2*CL*g01*g03*g05*gm2*gm4*gm8 + 2*CL*g01*g03*g06*gm2*gm4*gm7 +
2*CL*g01*g03*g08*gm2*gm4*gm5 + 2*CL*g01*g02*g06*gm3*gm5*gm7 + 2*CL*g02*g03*g06*gm1*gm5*gm7 +
2*CL*g02*g05*g06*gm1*gm3*gm5 + 2*CL*g02*g06*g07*gm1*gm3*gm5 + 2*CL*g01*g02*g03*gm4*gm7*gm8 +
2*CL*g01*g02*g06*gm4*gm5*gm7 + 2*CL*g01*g03*g04*gm2*gm7*gm8 + 2*CL*g01*g03*g07*gm2*gm4*gm8 +
2*CL*g01*g03*g08*gm2*gm4*gm5 + 2*CL*g01*g05*g06*gm2*gm5*gm7 + 2*CL*g01*g05*g06*gm2*gm4*gm7 +
2*CL*g01*g06*g07*gm2*gm4*gm5 + 2*CL*g01*g02*g03*gm5*gm7*gm8 + 2*CL*g01*g02*g05*gm3*gm7*gm8 +
2*CL*g01*g02*g07*gm3*gm5*gm8 + 2*CL*g01*g02*g08*gm3*gm5*gm7 + 2*CL*g01*g03*g06*gm4*gm5*gm7 +
2*CL*g01*g04*g06*gm3*gm5*gm7 + 2*CL*g01*g05*g06*gm3*gm4*gm7 + 2*CL*g01*g06*g07*gm3*gm4*gm5 +
2*CL*g02*g03*g05*gm1*gm7*gm8 + 2*CL*g02*g03*g07*gm1*gm5*gm8 + 2*CL*g02*g03*g08*gm1*gm5*gm7 +
2*CL*g02*g05*g07*gm1*gm3*gm8 + 2*CL*g02*g05*g08*gm1*gm3*gm7 + 2*CL*g02*g07*g08*gm1*gm3*gm5 +
2*CL*g03*g04*g06*gm1*gm5*gm7 + 2*CL*g03*g05*g06*gm1*gm4*gm7 + 2*CL*g03*g06*g07*gm1*gm4*gm5 +
2*CL*g04*g06*g07*gm1*gm3*gm5 + 2*CL*g04*g06*g07*gm1*gm3*gm4 + 2*CL*g05*g06*g07*gm1*gm3*gm4 +
2*CL*g04*g05*g06*gm1*gm3*gm7 + 2*CL*g04*g05*g06*gm1*gm3*gm5 + 2*CL*g04*g07*gm1*gm3*gm5 +
2*CL*g04*g08*gm1*gm3*gm5 + 2*CL*g04*g08*gm1*gm3*gm4 + 2*CL*g05*g07*gm1*gm3*gm4 +
2*CL*g05*g07*gm1*gm3*gm5 + 2*CL*g05*g08*gm1*gm3*gm4 + 2*CL*g05*g08*gm1*gm3*gm5 +
2*CL*g06*g07*gm1*gm3*gm4 + 2*CL*g06*g07*gm1*gm3*gm5 + 2*CL*g07*gm1*gm3*gm4 +
2*CL*g07*gm1*gm3*gm5 + 2*CL*g08*gm1*gm3*gm4 + 2*CL*g08*gm1*gm3*gm5 +
2*CL*g03*g05*gm1*gm4*gm7*gm8 + 2*CL*g03*g07*gm1*gm4*gm5*gm8 + 2*CL*g03*g08*gm1*gm4*gm5*gm7 +
2*CL*g04*g05*gm1*gm3*gm7*gm8 + 2*CL*g04*g07*gm1*gm3*gm5*gm8 + 2*CL*g04*g08*gm1*gm3*gm5*gm7 +
2*CL*g05*g07*gm1*gm3*gm4*gm8 + 2*CL*g05*g07*gm1*gm3*gm5*gm7 + 2*CL*g06*g07*gm1*gm3*gm4*gm5 +
2*CL*g06*g07*gm1*gm3*gm5*gm7 + 2*CL*g07*gm1*gm3*gm4*gm5 + 2*CL*g07*gm1*gm3*gm5*gm7 +
2*CL*g08*gm1*gm3*gm4*gm5 + 2*CL*g08*gm1*gm3*gm5*gm7 +
2*CL*g03*g05*gm2*gm4*gm7*gm8 + 2*CL*g03*g07*gm2*gm4*gm5*gm8 + 2*CL*g03*g08*gm2*gm4*gm5*gm7 +
2*CL*g04*g05*gm2*gm3*gm7*gm8 + 2*CL*g04*g07*gm2*gm3*gm5*gm8 + 2*CL*g04*g08*gm2*gm3*gm5*gm7 +
2*CL*g05*g07*gm2*gm3*gm4*gm8 + 2*CL*g05*g07*gm2*gm3*gm5*gm7 + 2*CL*g06*g07*gm2*gm3*gm4*gm5 +
2*CL*g06*g07*gm2*gm3*gm5*gm7 + 2*CL*g07*gm2*gm3*gm4*gm5 + 2*CL*g07*gm2*gm3*gm5*gm7 +
2*CL*g08*gm2*gm3*gm4*gm5 + 2*CL*g08*gm2*gm3*gm5*gm7 +
2*CL*g03*g05*gm1*gm4*gm7*gm8 + 2*CL*g03*g07*gm1*gm4*gm5*gm8 + 2*CL*g03*g08*gm1*gm4*gm5*gm7 +
2*CL*g04*g05*gm1*gm3*gm7*gm8 + 2*CL*g04*g07*gm1*gm3*gm5*gm8 + 2*CL*g04*g08*gm1*gm3*gm5*gm7 +
2*CL*g05*g07*gm1*gm3*gm4*gm8 + 2*CL*g05*g07*gm1*gm3*gm5*gm7 + 2*CL*g06*g07*gm1*gm3*gm4*gm5 +
2*CL*g06*g07*gm1*gm3*gm5*gm7 + 2*CL*g07*gm1*gm3*gm4*gm5 + 2*CL*g07*gm1*gm3*gm5*gm7 +
2*CL*g08*gm1*gm3*gm4*gm5 + 2*CL*g08*gm1*gm3*gm5*gm7 +
2*CL*g03*g05*gm1*gm4*gm7*gm8 + 2*CL*g03*g07*gm1*gm4*gm5*gm8 + 2*CL*g03*g08*gm1*gm4*gm5*gm7 +
2*CL*g04*g05*gm1*gm3*gm7*gm8 + 2*CL*g04*g07*gm1*gm3*gm5*gm8 + 2*CL*g04*g08*gm1*gm3*gm5*gm7 +
2*CL*g05*g07*gm1*gm3*gm4*gm8 + 2*CL*g05*g07*gm1*gm3*gm5*gm7 + 2*CL*g06*g07*gm1*gm3*gm4*gm5 +
2*CL*g06*g07*gm1*gm3*gm5*gm7 + 2*CL*g07*gm1*gm3*gm4*gm5 + 2*CL*g07*gm1*gm3*gm5*gm7 +
2*CL*g08*gm1*gm3*gm4*gm5 + 2*CL*g08*gm1*gm3*gm5*gm7 +
```

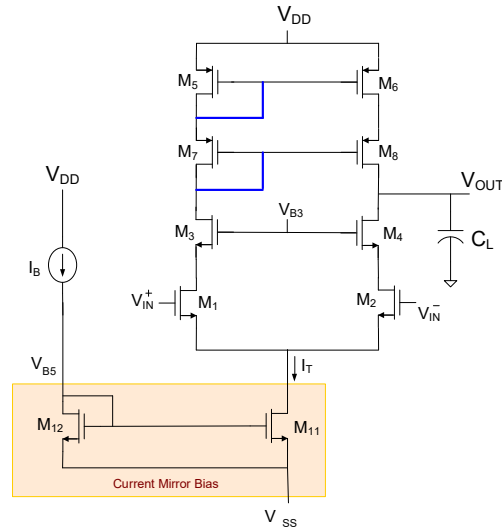

MATLAB Solution Continued 5:

```
2*g01*g02*g04*g06*g011*gm5*gm7 + 2*g01*g02*g04*g05*g011*gm7*gm8 + 2*g01*g02*g04*g07*g011*gm5*gm8
+ 2*g01*g02*g04*g08*g011*gm5*gm7 + 2*g01*g03*g06*g08*g011*gm4*gm5 +
2*g02*g03*g04*g06*g011*gm5*gm7 + 2*g02*g04*g05*g06*g011*gm3*gm7 + 2*g02*g04*g06*g07*g011*gm3*gm5
+ 2*g01*g02*g06*g08*g011*gm5*gm7 + 2*g01*g03*g06*g08*g011*gm4*gm7 +
2*g02*g03*g04*g05*g011*gm7*gm8 + 2*g02*g03*g04*g07*g011*gm5*gm8 + 2*g02*g03*g04*g08*g011*gm5*gm7
+ 2*g02*g04*g05*g07*g011*gm3*gm8 + 2*g02*g04*g05*g08*g011*gm3*gm7 +
2*g02*g04*g07*g08*g011*gm3*gm5 + 2*g01*g04*g06*g08*g011*gm5*gm7 + 2*g01*g05*g06*g08*g011*gm4*gm7
+ 2*g01*g06*g07*g08*g011*gm4*gm5 + 2*g02*g03*g05*g06*g08*g011*gm5*gm7 +
2*g02*g05*g06*g08*g011*gm3*gm7 + 2*g02*g06*g07*g08*g011*gm3*gm5 + 2*g03*g04*g06*g08*g011*gm5*gm7
+ 2*g03*g05*g06*g08*g011*gm4*gm7 + 2*g03*g06*g07*g08*g011*gm4*gm5 + 2*g05*g06*g07*g08*g011*gm3*gm4
+ 2*g01*g02*g04*g06*gm3*gm5*gm7 + 2*g02*g03*g04*g06*gm1*gm5*gm7 + 2*g02*g04*g05*g06*gm1*gm3*gm7
+ 2*g02*g04*g06*g07*gm1*gm3*gm5 + 2*g01*g02*g04*g06*gm3*gm5*gm8 + 2*g01*g03*g06*g08*gm2*gm4*gm5
+ 2*g02*g03*g04*g05*gm1*gm7*gm8 + 2*g02*g03*g04*g07*gm1*gm5*gm8 + 2*g02*g03*g04*g08*gm1*gm5*gm7
+ 2*g02*g04*g05*g07*gm1*gm3*gm8 + 2*g02*g04*g05*g08*gm1*gm3*gm7 + 2*g02*g04*g07*g08*gm1*gm3*gm5
+ 2*g01*g02*g04*g06*gm3*gm7*gm8 + 2*g01*g03*g06*g08*gm2*gm4*gm7 + 2*g02*g03*g04*g06*gm1*gm7*gm8
+ 2*g02*g04*g06*g07*gm1*gm3*gm8 + 2*g01*g02*g06*g08*gm3*gm5*gm7 + 2*g02*g03*g06*g08*gm1*gm5*gm7
+ 2*g02*g05*g06*g08*gm1*gm3*gm7 + 2*g02*g05*g06*g08*gm2*gm5*gm7 + 2*g01*g05*g06*g08*gm2*gm4*gm7
+ 2*g01*g06*g07*g08*gm2*gm4*gm5 + 2*g02*g03*g04*g08*gm1*gm7*gm8 + 2*g02*g04*g07*g08*gm1*gm3*gm8
+ 2*g01*g03*g06*g08*gm4*gm5*gm7 + 2*g01*g04*g06*g08*gm3*gm5*gm7 + 2*g01*g05*g06*g08*gm3*gm4*gm7
+ 2*g01*g06*g07*g08*gm3*gm4*gm5 + 2*g03*g04*g06*g08*gm1*gm5*gm7 + 2*g03*g05*g06*g08*gm1*gm4*gm7
+ 2*g03*g06*g07*g08*gm1*gm4*gm5 + 2*g04*g05*g06*g08*gm1*gm3*gm7 + 2*g04*g06*g07*g08*gm1*gm3*gm5
+ 2*g05*g06*g07*g08*gm1*gm3*gm4 + 2*g02*g03*g06*g08*gm4*gm5*gm7 + 2*g02*g06*g07*g08*gm3*gm4*gm5
+ 2*g02*g06*g07*g08*gm2*gm4*gm7 + 2*g02*g06*g07*g08*gm2*gm4*gm5 + 2*g03*g04*g06*g08*gm2*gm5*gm7
+ 2*g04*g06*g07*g08*gm2*gm3*gm5 + 2*g05*g06*g07*g08*gm2*gm3*gm4 + 2*g01*g02*g04*g011*gm5*gm7*gm8
+ 2*g02*g04*g05*g011*gm3*gm5*gm7 + 2*g02*g03*g04*g011*gm5*gm7*gm8 +
2*g02*g04*g05*g011*gm3*gm7*gm8 + 2*g02*g04*g07*g011*gm3*gm5*gm8 + 2*g02*g04*g08*g011*gm3*gm5*gm7
+ 2*g01*g06*g08*g011*gm4*gm5*gm7 + 2*g02*g06*g08*g011*gm3*gm5*gm7 +
2*g03*g06*g08*g011*gm4*gm5*gm7 + 2*g04*g06*g08*g011*gm3*gm5*gm7 + 2*g05*g06*g08*g011*gm3*gm4*gm7
+ 2*g06*g07*g08*g011*gm3*gm4*gm5 + 2*g02*g04*g06*gm1*gm3*gm5*gm7 + 2*g02*g04*g06*gm1*gm3*gm5*gm8
+ 2*g01*g02*g04*gm3*gm5*gm7*gm8 + 2*g02*g03*g04*gm1*gm5*gm7*gm8 + 2*g02*g04*g05*gm1*gm3*gm7*gm8
+ 2*g02*g04*g06*gm1*gm3*gm5*gm7 + 2*g02*g04*g06*gm1*gm3*gm7*gm8 + 2*g02*g04*g08*gm1*gm3*gm7*gm8
+ 2*g01*g06*g08*gm3*gm4*gm5*gm7 + 2*g03*g06*g08*gm1*gm4*gm5*gm7 + 2*g04*g06*g08*gm1*gm3*gm5*gm7
+ 2*g05*g06*g08*gm1*gm3*gm4*gm5 + 2*g06*g07*g08*gm1*gm3*gm4*gm5 + 2*g02*g06*g08*gm3*gm4*gm5*gm7
+ 2*g03*g06*g08*gm2*gm4*gm5*gm7 + 2*g04*g06*g08*gm2*gm3*gm5*gm7 + 2*g05*g06*g08*gm2*gm3*gm4*gm7
+ 2*g06*g07*g08*gm2*gm3*gm4*gm5 + 2*g02*g04*g011*gm3*gm5*gm7*gm8 +
2*g06*g08*g011*gm3*gm4*gm5*gm7 + 2*g02*g04*g04*gm3*gm5*gm7*gm8 + 2*g06*g08*gm1*gm3*gm4*gm5*gm7 +
2*g06*g08*gm2*gm3*gm4*gm5*gm7 )
```

- Difference Mode Gain has only approximately 1100 product terms
- Difference Mode Gain has approximately 7700 small-signal parameters in expression
 - Extremely difficult to get insight into how this relatively simple circuit performs from this solution
- This does not even include the common-mode gain expression !

Where this started

Telescopic Cascode Op Amp with Mirror-connected Counterpart Circuit

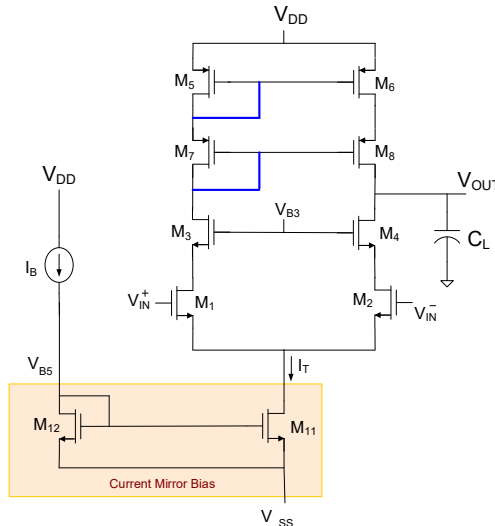


- Previous analysis obtained almost by inspection
- Gain expressions were real simple
- Gain expressions provide major insight into operation
- Some assumptions were made to simplify analysis
 - $V_{ac}=0$ at “approximate axis of symmetry”
 - Matched left and right side transistors
 - Current mirror used to mirror left-side current to right side

How much more involved would an exact analysis be ?
Would an exact analysis provide additional insight ?

Where this started

Telescopic Cascode Op Amp with Mirror-connected Counterpart Circuit



$$A_d(s) = \frac{-g_{m1}}{sC_L + g_{o1} \frac{g_{o3}}{g_{m3}} + g_{o5} \frac{g_{o7}}{g_{m7}}}$$

$$A_d(s) = \frac{-g_{m7}g_{m1}g_{m3}}{sC_L g_{m7}g_{m3} + g_{m7}g_{o3}g_{o1} + g_{m3}g_{o5}g_{o7}}$$

- Some assumptions were made to simplify analysis
 - $V_{ac}=0$ at “approximate axis of symmetry”
 - Matched left and right side transistors
 - Current mirror used to mirror left-side current to right side
- Difference Mode Gain has only approximately 1100 product terms
- Difference Mode Gain has approximately 7700 small-signal parameters in expression

How many product terms are present in the simplified analysis?

How many small-signal parameters are in simplified expression?

- Simplified Difference Mode Gain has 4 product terms
- Simplified Difference Mode Gain has 12 small-signal parameters in expression

Be careful about what you ask for, you may be able to get it !

- How important is it to develop good approximate analysis methods for an op amp of this complexity?
- And many useful op amp circuits will have much more complexity !!

Will you impress your boss if you use an “exact” analysis?





Stay Safe and Stay Healthy !

End of Lecture 7